MAU23205 2021-2022 Practice Problem Set 3

1. Verify that

$$W(t,s) = \begin{bmatrix} \cos(\log(t/s)) & s\sin(\log(t/s)) \\ -\frac{1}{t}\sin(\log(t/s)) & \frac{s}{t}\cos(\log(t/s)) \end{bmatrix}$$

is the fundamental solution for

$$A(t) = \begin{bmatrix} 0 & 1 \\ -\frac{1}{t^2} & -\frac{1}{t} \end{bmatrix}$$

in the sense of Lecture 10.

2. Express the solution to the initial value problem $x(t_0) = x_0$ $x'(t_0) = y_0$ for the differential equation

$$t^2x''(t) + tx'(t) + x(t) = f(t)$$

in terms of t_0, x_0, y_0 and f.
Note: You may use the previous problem, even if you didn't manage to solve it. Your solution will involve an integral. Rewatch Lecture 9 if you don't know where to start.

3. Find a set of basic solutions to the equation

$$x^{(7)} + 3x^{(6)} + 5x^{(5)} + 7x^{(4)} + 7x''' + 5x'' + 3x' + x = 0.$$

Note:

$$z^7 + 3z^6 + 5z^5 + 7z^4 + 7z^3 + 5z^2 + 3z + 1 = (z+1)^3(z^2+1)^2.$$