MAU23205 2021-2022 Practice Problem Set 2

1. The existence and uniqueness theorem shows that the initial value problem

$$x(t_0) = x_0 \qquad y(t_0) = y_0 \qquad z(t_0) = z_0$$

for the system

$$x'(t) = y(t)z(t) \qquad y'(t) = -x(t)z(t) \qquad z'(t) = -k^2x(t)z(t)$$

has, for each k, a unique solution which depends continuously not only on t but also on the initial values t_0, x_0, y_0 and z_0. Explain how to use the existence and uniqueness theorem to get continuous dependence on k as well.
Hint: Apply it to a larger system.

2. Find numerical approximations to the system

$$x'(t) = -y(t) \qquad y'(t) = x(t)$$

using the second order predictor-corrector method instead of Euler. Do the approximate solutions remain bounded at t tends to infinity for a fixed step size h?

3. Suppose that A is an $n \times n$ matrix.
   (a) Show that A and A^T have the same characteristic polynomial.
   (b) Show that A and A^T have the same minimal polynomial.