MAU23205 2021-2022 Practice Problem Set 1

1. Assume that $\alpha, \beta, \gamma, \delta > 0$. Show that

$$I = \left(\frac{\delta x}{\gamma}\right)^{-\sqrt{\gamma/\alpha}} \left(\frac{\beta y}{\alpha}\right)^{-\sqrt{\alpha/\gamma}} \exp\left(\frac{\delta x + \beta y}{\sqrt{\alpha\gamma}}\right)$$

   is an invariant of the system

$$\frac{dx}{dt} = \alpha x - \beta xy \qquad \frac{dy}{dt} = -\gamma y + \delta xy$$

2. The differential equation

$$\frac{dy}{dx} = -\frac{6x + 2y + 5}{2x + 2y + 4}$$

   has a quadratic invariant, i.e. an invariant of the form

$$I = ax^2 + bxy + cy^2 + dx + ey + f.$$

   (a) In fact there are infinitely many quadratic invariants, but find at least one non-zero invariant.

   (b) Use this invariant to solve the initial value problem

$$y(0) = -4.$$