MAU23205 2021-2022 Assignment 2 Due 12 November 2021

1. Find $\exp(tA)$ where

$$A = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -2 & -1 & 3 \\ 4 & -3 & -1 & 5 \\ 3 & -3 & -1 & 4 \end{bmatrix}.$$

Hint: The characteristic polynomial is $z^4 + 2z^2 + 1$.

2. Solve the boundary value problem

$$y(-L) = 0 = y(L)$$

for the differential equation

$$y''(x) + \omega^2 y(x) = \cos(x),$$

where $L > 0$ and $\omega > 0$.
Hint: Depending on the values of L and ω there may be a unique solution, no solution, or infinitely many solutions.

3. Do the all the solutions to

$$\frac{1}{6}x'''(t) + \frac{1}{2}x''(t) + x'(t) + x(t) = 0$$

tend to zero as t tends to $+\infty$?