MAU23205 2021-2022 Assignment 1 Due 11 October 2021

1. The system

$$\frac{dx}{dt} = x - y - x^2 - 2xy - y^2 \qquad \frac{dy}{dt} = x - y + x^2 + 2xy + y^2$$

has an invariant which is a cubic polynomial in x and y. Find it.

2. The differential equation

$$p''(z) = 6p(z)^2 - \frac{1}{2}g_2$$

arises in the theory of elliptic functions. Do not attempt to solve it. Instead use the general existence and uniqueness theorem to show that solutions to the initial value problem $p(z_0) = p_0$, $p'(z_0) = q_0$ depend continuously on p_0, q_0 *and* g_2.

Hint: This requires replacing the equation with an appropriate system.

3. Rewrite
$$tx''(t) + (1 - t)x'(t) + \lambda x(t) = f(t)$$

as $\mathbf{x}'(t) = A(t)\mathbf{x}(t) + \mathbf{g}(t)$ for appropriate A and $\mathbf{g}$.