MAU22C00 Assignment 8, Due Friday 24 November 2023 Solutions

1. Find all the monoid homomorphisms from (N, +), i.e. the monoid whose set is natural numbers and whose operation is addition, to itself. *Hint:* Describe them in terms of what value the function takes at 1. So*lution:* Suppose h is a homomorphism. Then $h(m) = m \cdot h(1)$. This is easily proved by induction. h(0) = 0 and if $h(m) = m \cdot h(1)$ then

$$h(m+1) = h(m) + h(1) = m \cdot h(1) + h(1) = (m+1) \cdot h(1).$$

So all homomorphisms must be of the form $h(m) = m \cdot a$ for some $a \in N$. Conversely, if h is of this form then it is a semigroup homomorphism because

$$h(j+k) = (j+k) \cdot a = j \cdot a + k \cdot a = h(j) + h(k).$$

It's a monoid homomorphism because, in addition,

$$h(0) = 0 \cdot a = 0$$

2. Give a left regular grammar and a right regular grammar for the language of strings of x's and y's. In other words a string should be generated by the grammar if and only if every character in it is an x or a y. This is vacuously true of the empty string, so it should be generated by your language. *Solution:* There are multiple possibilities but a left regular grammar is

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start : xy ;
xy : | xy x | xy y ;

and a right regular grammar is

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start : xy ;
xy : | x xy | y xy ;
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3. Give a left regular grammar for the language of those strings of x's, y's or z's which have no occurence of x, no occurence of y, or no occurence of z. In case it's not obvious, by this I mean that there is at least one of the letters, x, y, or z which does not appear in the string; I don't mean that none of them appear. *Solution:* Such a string must belong to the language of strings of x's and y's, which we just found a grammar for, or the language of strings of x's and z's or of y'z and z's, which are easy modifications of it. In this way we get the grammar

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start : xy | xz | yz ;
xy : | xy x | xy y ;
xz : | xz x | xz z ;
yz : | yz y | yz z ;