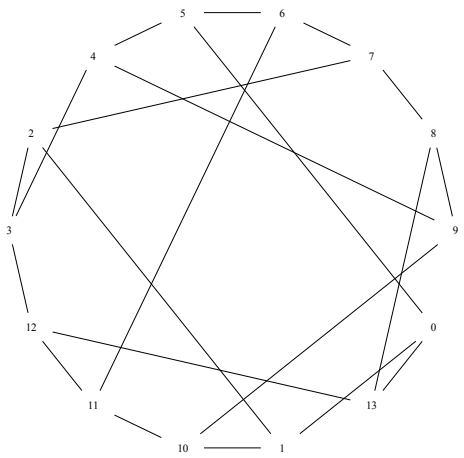
MAU22C00 Assignment 6, Due Friday 10 November 2023
Solutions

1. Consider a line in the plane with two points marked on it. We'll say that a point is constructible if there is some Eulidean geometry construction which allows you to find that point, starting from that line and those points. Show that there are points on the line which are not constructible.
Note: You don't need much knowledge of Euclidean geometry for this problem. You can use the fact that points on a line can be parameterised by real numbers.
Solution: You can write down Euclidean geometry constructions, i.e. for each one there is corresponding string of characters. These are lists items chosen from a finite set so the set of such constructions is countable. The set of points on the line is the same as the number of real numbers and hence is uncountable. So there must be a point which is not constructible.

2. Consider the following undirected graph.



Is it

(a) bipartite?
(b) complete?
(c) connected?
(d) directed?
(e) regular?

Hint: The numbering of the vertices is not random.

Solution:

(a) Yes. If you look at the diagram you can see that the only edges go from even numbered vertices to odd numbered vertices.

(b) No. There are pairs of vertices with no edge between them, e.g. 0 and 1.

(c) Yes. If you look at the diagram you can see that a path from any vertex to any other vertex is obtained by visiting all the numbers in between in order.

(d) Yes. All undirected graphs are directed graphs. The terminology is confusing, but you have to learn to live with that.

(e) Yes. Every vertex degree 3.

3. Does the graph in the preceding problem have

   (a) A Hamiltonian path?

   (b) An Eulerian trail?

   Solution:

   (a) Yes. In fact there are a lot of them–168, to be exact–but the one I'm expecting people to notice is $(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13$.

   (b) No. A necessary condition for the existence of an Eulerian trail is that the number of vertices of odd order is at most two. Here it's 14.