MAU22C00 Assignment 5, Due Friday 3 November 2023

1. Show that if A and B are finite sets then so is $A \times B$.
   Hint: Use induction on sets, twice. Let C be the set of subsets D of A such that $D \times B$ is finite. Show that $\emptyset \in C$ and that if $D \in C$ and $x \in A$ then $D \bigcup \{x\} \in C$. For the second of these it's helpful to prove that $\{x\} \times B$ is finite. For that, let E be the set of subsets F of B such that $\{x\} \times F$ is finite. Show that $\emptyset \in E$ and that if $F \in E$ and $y \in B$ then $F \bigcup \{y\} \in E$.

2. Suppose A is a set and $B = PA$ is its power set. Let C be the set of ordered pairs (D, E) of members of B such that $D \subseteq E$. Is the binary relation C on B

   (a) left total?

   (b) right total?

   (c) left unique?

   (d) right unique?

   Some of the answers could depend on A.

3. There is a proof in the notes that N^3 is countable, but unlike the proof given there for N^2 it doesn't give an actual injective function to N. Given an example of an injective function from N^3 to N.
   Hint: You can use the same idea as for N^2, although the picture given there is harder to draw in three dimensions.