MAU22C00 Assignment 4, Due Friday 20 October 2023

1. State the following in the formal language of elementary arithmetic:
   (a) the commutative law for addition
   (b) the associative law for multiplication
   (c) the distributive law
   (d) that there are infinitely many odd numbers

2. Give an informal proof that the union of two arithmetic sets or the intersection of two arithmetic sets is arithmetic.

3. Which axiom or rule of inference is being used on each line of the following proof of $[(0'' \cdot 0'') = 0'''']$?

1. $\{\forall x.[(x \cdot 0) = 0]\}$
2. $[(0'' \cdot 0) = 0]$
3. $[(0'' \cdot 0)' = 0']$
4. $[(0'' \cdot 0)'' = 0'']$
5. $[\forall x.(\forall y.\{(x \cdot y') = [(x \cdot y) + x]\})]$
6. $(\forall y.\{(0'' \cdot y') = [(0'' \cdot y) + 0'']\})$
7. $\{(0'' \cdot 0') = [(0'' \cdot 0) + 0'']\}$
8. $(\forall x.\{\forall y.[(x + y') = (x + y)']\})$
9. $(\forall y.\{[(0'' \cdot 0) + y'] = [(0'' \cdot 0) + y]'\})$
10. $\{[(0'' \cdot 0) + 0''] = [(0'' \cdot 0) + 0']'\})$
11. $\{(0'' \cdot 0') = [(0'' \cdot 0) + 0']'\}$
12. $\{[(0'' \cdot 0) + 0'] = [(0'' \cdot 0) + 0]'\}$
13. $(\forall x.\{[x + 0] = x\})$
14. $\{[(0'' \cdot 0) + 0] = (0'' \cdot 0)\}$
15. $\{[(0'' \cdot 0) + 0] = 0\}$
16. $\{[(0'' \cdot 0) + 0]' = 0'\}$
17. $\{[(0'' \cdot 0) + 0'] = 0'\}$
18. $\{[(0'' \cdot 0) + 0']' = 0''\}$
19. $\{[(0'' \cdot 0) + 0''] = 0''\}$
20. $[(0'' \cdot 0') = 0'']$
21. $\{(0'' \cdot 0'') = [(0'' \cdot 0') + 0'']\}$
22. $(\forall y.\{[(0'' \cdot 0') + y'] = [(0'' \cdot 0') + y]'\})$
23. $\{[(0'' \cdot 0') + 0''] = [(0'' \cdot 0') + 0']'\}$
24. $\{(0'' \cdot 0'') = [(0'' \cdot 0') + 0']'\}$
25. $\{[(0'' \cdot 0') + 0'] = [(0'' \cdot 0') + 0]'\})$
26. $\{[(0'' \cdot 0') + 0] = (0'' \cdot 0')\}$
27. $\{[(0'' \cdot 0') + 0] = 0''\}$
28. $\{[(0'' \cdot 0') + 0]' = 0'''\}$
29. $\{[(0'' \cdot 0') + 0'] = 0'''\}$
30. $\{[(0'' \cdot 0') + 0']' = 0''''\}$
31. $[(0'' \cdot 0'') = 0'''']$

For reference here are the axioms and rules of inference for arithmetic.

1 $\{\forall x.[\neg(x' = 0)]\}$

2 $\{\forall x.[(x + 0) = x]\}$

3 $(\forall x.\{\forall y.[(x + y') = (x + y)']\})$

4 $\{\forall x.[(x \cdot 0) = 0]\}$

5 $[\forall x.(\forall y.\{(x \cdot y') = [(x \cdot y) + x]\})]$

1 From a statement of the form $(X = Y)$ we can derive $(Y = X)$.

2 From statements of the form $(X = Y)$ and $(Y = Z)$ we can derive $(X = Z)$.

3 From a statement of the form $(X = Y)$ we can derive $(X' = Y')$.

4 From a statement of the form $(X' = Y')$ we can derive $(X = Y)$.

5 Suppose Q is the Boolean expression P with all free occurences of v replaced by 0 and R is P with all free occurences of v replaced by v'. From Q and $[\forall v.(P \supset R)]$ we can derive $(\forall v.P)$. The same holds with any other variable in place of v.

You can also use the quantifier rules from first order logic.