MAU22C00 Assignment 2, Due Friday 4 October 2023

1. Write down a tableau to show that the statement $[(p \supset q) \supset (q \supset p)]$ is satisfiable and another to show that it is not a tautology.

2. Take the following proof in the natural deduction system and indicate which rule of inference is being used in each line.

$$\begin{array}{lll}
1 & . & [p \wedge (\neg p)] \\
2 & . & p \\
3 & . & (\neg p) \\
4 & . & .\quad (\neg q) \\
5 & . & .\quad [\neg(\neg p)] \\
6 & . & \{(\neg q) \supset [\neg(\neg p)]\} \\
7 & . & [(\neg p) \supset q] \\
8 & . & q \\
9 & \multicolumn{2}{l}{\{[p \wedge (\neg p)] \supset q\}}
\end{array}$$

The line numbering isn't technically part of the proof; I've just added it to make it easier to refer to individual lines.

3. The following proof of $\{[(p \vee r) \wedge (\neg(p \vee r))] \supset (r \supset q)\}$ is the same as the one above, except for the last two lines, which use the rule of substitution.

$$\begin{array}{lll}
1 & . & [p \wedge (\neg p)] \\
2 & . & p \\
3 & . & (\neg p) \\
4 & . & .\quad (\neg q) \\
5 & . & .\quad [\neg(\neg p)] \\
6 & . & \{(\neg q) \supset [\neg(\neg p)]\} \\
7 & . & [(\neg p) \supset q] \\
8 & . & q \\
9 & \multicolumn{2}{l}{\{[p \wedge (\neg p)] \supset q\}} \\
10 & \multicolumn{2}{l}{\{[p \wedge (\neg p)] \supset (r \supset q)\}} \\
11 & \multicolumn{2}{l}{\{[(p \vee r) \wedge (\neg(p \vee r))] \supset (r \supset q)\}}
\end{array}$$

Give an alternate proof without substitution. What property of the natural deduction system makes this possible? Can the same be done with all proofs which use the rule of substitution?
*Hint:* Instead of substituting at the end you can run the same argument but with all the substitutions done from the start.