MAU22200 2021-2022 Practice Problem Set 5

1. Suppose $(X, \mathcal{T}_X)$ is a topological space, $A \in \wp(X)$ and $\mathcal{T}_A$ is the subspace topology on A.

   (a) Show that if $U \in \mathcal{T}_X$ and $U \subseteq A$ then $U \in \mathcal{T}_A$.

   (b) Show that if $X \setminus V \in \mathcal{T}_X$ and $V \subseteq A$ then $A \setminus V \in \mathcal{T}_A$.

   (c) Show that if $A \in \mathcal{T}_X$, $U \in \mathcal{T}_A$ and $U \subseteq A$ then $U \in \mathcal{T}_X$.

   (d) Show that if $X \setminus A \in \mathcal{T}_X$, $A \setminus V \in \mathcal{T}_A$ and $V \subseteq A$ so $X \setminus V \in \mathcal{T}_X$ because intersections of open sets are open.

2. The cofinite topology on a set X was defined in Practice Problem Set 2, where you proved that it is indeed a topology and that it is a Hausdorff topology if and only if X is finite.

   Proposition 3.10.9 says that $(X, \mathcal{T})$ is Hausdorff if and only if the diagonal Δ_X is closed. If X is infinite then it follows that Δ_X is not closed, so by Proposition 3.2.2 Parts (b) and (g) the closure of Δ_X is strictly larger than Δ_X. What is the closure of Δ_X?

3. (a) Suppose $A \in \wp(\mathbf{R})$ is connected. First show that if $x < y < z$ and $x, z \in A$ then $y \in A$.

   (b) Show that if A is connected then A is an interval.