MAU22200 2021-2022 Practice Problem Set 4

1. Suppose that X and Y are non-empty sets. Show that there is a surjection $f\colon X \to Y$ or a surjection $g\colon Y \to X$.
   Hint: While it is possible to prove this using Zorn's Lemma, I don't really recommend using it directly. It's considerably easier to use results you already have from the notes.

2. Which of the following are countable? Justify your answers.

   (a) The set of subsets of the set of rational numbers.

   (b) The set of empty subsets of the set of rational numbers.

   (c) The set non-empty subsets of the set of rational numbers.

   (d) The set finite subsets of the set of rational numbers.

3. (a) Find A°, $\overline{(A^\circ)}$ and $\left(\overline{(A^\circ)}\right)^\circ$ where

$$A = (-1,0) \cup (0,1)$$

   (b) Find $\overline{A}$ and $\left(\overline{A}\right)^\circ$ where

$$A = \mathbf{Q} \cap [-1,1]$$

4. (a) Show that
$$A^\circ \subseteq \left(\overline{(A^\circ)}\right)^\circ.$$

   (b) Show that
$$\overline{\left(\left(\overline{A}\right)^\circ\right)} \subseteq \overline{A}.$$

   (c) Show that
$$\overline{(A^\circ)} = \overline{\left(\left(\overline{(A^\circ)}\right)^\circ\right)}.$$