MAU22200 2021-2022 Practice Problem Set 3

1. For any set X and any $x \in X$ the set

$$\mathcal{F} = \{A \in \wp(X) \colon x \in A\}$$

is called the principal filter of X at x. Show that it is indeed a filter.

2. Show that the neighbourhood filter $\mathcal{N}(x)$ is a subset of the principal filter at x.

3. Suppose X is infinite. The cofinite filter on X is defined to be the set of subsets of A of X such that $X \setminus A$ is finite. Show that it is indeed a filter.

4. Show that the cofinite filter is not contained in any principal filter.

5. List all the filters on the set $\{1, 2, 3\}$.