MAU22200 2021-2022 Practice Problem Set 2

1. Lemma 1.9.4 in the notes gives the following two inclusions of sets. Give an example in each case to show that the inclusion cannot be replaced by equality in general.

   (a) $\varphi_*(A \cap B) \subseteq \varphi_*(A) \cap \varphi_*(B)$.

   (b) $\varphi_*(A \setminus B) \supseteq \varphi_*(A) \setminus \varphi_*(B)$.

2. Suppose $(X, \mathcal{T})$ is a topological space. Show that $\mathcal{T}$ is Hausdorff if and only if for all distinct $x, y \in X$ there are $P \in \mathcal{N}(x)$ and $Q \in \mathcal{N}(y)$ such that $P \cap Q = \varnothing$.

3. If X is a set then the *cofinite* topology on X is the set $\mathcal{T}$ consisting of those $U \in \wp(X)$ such that $U = \varnothing$ or $X \setminus U$ is finite.

   (a) Show that $\mathcal{T}$ is indeed a topology on X.

   (b) Show that $\mathcal{T}$ is Hausdorff if and only if X is finite.