MAU22200 2021-2022 Practice Assignment 2, Due 1 December 2021

1. For each of the following subsets of $\mathbf{R}$ answer each of the following questions: Is it open? Is it closed? Is it Hausdorff? Is it connected? Is it compact? Is it bounded?
   Note: You're only asked to provide yes or no answers, not proofs.

   (a) $A = (-2, 1] \cup [-1, 2)$

   (b) $B = (-\infty, -1] \cup [1, +\infty)$

   (c) $C = \mathbf{Q} \cap [-1, 1]$

   (d) $D = (-2, 2) \cap [-1, 1]$.

2. Suppose $A \subseteq B \subseteq C$ and $\mathcal{T}_C$ is a topology on C. Let $\mathcal{T}_B$ be the subspace topology on B as a subset of C and let $\mathcal{T}_A$ be the subspace topology on A as a subset of B. Is $\mathcal{T}_A$ also the subspace topology on A as a subset of C?

3. Suppose $(X, \mathcal{T})$ is a topological space For $x \in X$ let $\mathcal{S}(x)$ be the set of sets A such that $x \in A$, $A \subseteq X$ and A is connected. Let

$$B(x) = \bigcup_{A \in \mathcal{S}(x)} A.$$

   (a) Show that $x \in B(x)$ and that $B(x)$ is connected.

   (b) Show that for all $x, y \in X$ either $B(x) = B(y)$ or $B(x) \cap B(y) = \varnothing$.
   Hint: If $B(x) \cap B(y) = \varnothing$ then there's a $z \in B(x) \cap B(y)$. Try to show that $B(x) = B(z) = B(y)$.

   (c) Show that $B(x)$ is closed.
   Hint: Try showing that the closure of $B(x)$ is connected and contains x.

4. Suppose $(X, \mathcal{T})$ is a normal topological space and $K \subseteq U \subseteq X$ with K closed and U open. Show that there are closed L and open V such that $K \subseteq V \subseteq L \subseteq U$.