

Before we get to the problems, here are some general comments about how to use this practice exam:

- The actual exam has four questions, from which you are to choose three. The practice exam has seven questions, and so covers topics which we don't have time for on a two hour exam. I can't, of course, tell you which ones are on the real exam.
- On the real exam the number of points for each part of each question will be indicated. I haven't bothered to do that for the practice exam.
- For each topic there are a lot of different types of questions I could ask and if a topic does appear on the exam then that question could be of the same type as the one on the practice exam, but probably won't be. So the question you should be asking yourself when reading each question is not so much "Do I know how to do this particular problem?" as "Do I understand the relevant chapter well enough to do problems at a similar level of difficulty?" and if the answer is "no" then you need to revise the whole chapter, not the just the parts which are relevant to the problem from the sample exam.
- I won't be posting solutions to these problems because I fear that would encourage you to focus too much on these specific problems and not on the general topics, but if you have an answer and aren't sure whether it's correct you can email me and I will try to reply. I can't guarantee to reply to each question, particularly close to the date of the exam, because I don't know how many there will be.
- If you are someone who panics in exams it may be helpful during the real exam to remember that you don't actually have to pass this exam. You just need a passing result overall, on the continuous assessment combined with the exam. Last time I checked most people were doing quite well on the continuous assessment, i.e. the assignments, and they count for 30% of the overall mark so if you are one of those people then you are going in to the exam with a bit of an insurance policy against a bad exam score. Obviously you should still try to do as well as you can, but you shouldn't panic. Unless, of course, you haven't been doing the assignments, in which case you should probably panic now, before the exam starts.

1. (a) The following grammar is ambiguous.

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start : a
a      : b | b "+" b
b      : c | c "." c
c      : "x" | c c

```

Give an unambiguous grammar which generates the same language.

- (b) Does this grammar generate the same language?

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start : a
a      : b | b "." b
b      : c | c "+" c
c      : "x" | c c

```

Why or why not?

2. (a) Give a parse tree for the statement $\{[(\neg p) \vee p] \supset q\}$.
 (b) Determine, using a tableau, whether this statement is satisfiable.
 (c) Determine, using a tableau, whether this statement is a tautology.
3. (a) For each of the following four pairs of lines in first order logic, does the second line follow from the first?

i.

$$\begin{array}{l} [\forall x.(fxa)] \\ \{[\exists x.(x = a)] \supset (faa)\} \end{array}$$

ii.

$$\begin{array}{l} \{[\exists x.(x = a)] \supset (faa)\} \\ [\forall x.(fxa)] \end{array}$$

iii.

$$\begin{array}{l} [\exists x.(fxa)] \\ \{[\exists x.(x = a)] \wedge (faa)\} \end{array}$$

iv.

$$\begin{array}{l} \{[\exists x.(x = a)] \wedge (faa)\} \\ [\exists x.(fxa)] \end{array}$$

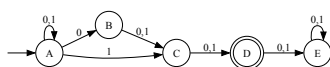
- (b) For any of the ones where the second line does not follow from the first, give an example to show why a rule of inference which allowed you to deduce the second from the first would be unsound.
4. (a) Write down a formal statement of the fact that 2 is the only even prime, in the formal language we used for elementary arithmetic.
 (b) Show that if A is an arithmetic set and F is finite then $A \setminus F$ is arithmetic.

5. The Cantor “middle thirds” set is what’s left after taking the interval $[0, 1]$ in the real line and removing its middle third, i.e. $(1/3, 2/3)$, then removing the middle third of each of the remaining two intervals, i.e. $(1/9, 2/9)$ and $(7/9, 8/9)$, then the middle third of each of the remaining four intervals, etc.

- (a) Show that the Cantor set is non-empty.
- (b) Show that the Cantor set is infinite.
- (c) Show that the Cantor set is uncountable.

Note: Each of these implies the previous ones, so if you’re sure your answer to the last one is correct you can skip the first two and if you’re sure your answer to the second one is correct you can skip the first one.

6. For the following finite state automaton



- (a) What is the start state or start states?
 - (b) What is the accepting state or accepting states?
 - (c) Is the input 0101 accepted?
 - (d) Give a regular grammar for the language of strings accepted by the automaton.
 - (e) Give a strongly deterministic finite state automaton which accepts the same set of inputs as this one.
7. (a) Describe a pushdown automaton which recognises the language consisting of some number of x ’s followed by an equal number of y ’s.
- (b) Check that your automaton correctly recognises xyy as a member of the language, by indicating the stack contents after each character is read.
- (c) Check that your automaton correctly recognises xyx and xyy as non-members of the language.