

MAU 22200 Week 6 Lecture 2

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Apologies

Sorry for not emailing out this week's problems until yesterday.

Terminology

Last time I said the setting for most of Section 1.4 was a set X , a set \mathcal{B} of subsets of X and a function μ from \mathcal{B} to $[0, +\infty]$ satisfying various conditions. These generalise the properties of \mathbf{R}^d , its set of measurable subsets and Lebesgue measure.

A triple (X, \mathcal{B}, μ) like this is called a *measure space*. The parts have names as well. Such a \mathcal{B} is called a σ -*algebra*. Such a μ is called a *measure*. Terry calls them countably additive (unsigned) measures for clarity.

There are also weaker notions, with countable replaced by finite. A set \mathcal{B} which is closed under complements and finite intersections or unions, but not necessarily countable ones is called a *Boolean algebra*. A function μ which is monotone and finitely additive, but not necessarily countably additive, is called a *finitely additive measure*. That includes the Jordan algebra and Jordan measure. They're less important though.

What's in Subsections 1.4.1 to 1.4.4?

Most of the first four subsections of Section 1.4 are a repetition of Sections 1.1 to 1.3 in a more general context, but some are not:

- ▶ There are new examples, e.g. trivial and discrete algebras, the null algebra, dyadic algebras, the Borel algebra, the Dirac measure, counting measure, etc. Some of these are just there as illustrations, but others are important.
- ▶ There are some new constructions, e.g. restriction of an algebra or measure, intersection of algebras, sums of measures, etc. The most important construction, products, is deferred to Section 1.7 though.
- ▶ There are some newish concepts and theorems. Those generalise constructions which appeared in special cases in earlier sections. For example, the Jordan algebra is generated by boxes, but this wasn't made explicit before.

What else is in Subsections 1.4.1 to 1.4.4?

- ▶ There are some new concepts, e.g. completeness of measure spaces.
- ▶ There are some new results, e.g. the results on Borel sets, or the ones on sequences of measures.
- ▶ Some theorems require additional hypotheses, e.g. Egorov's Theorem. The “generalised” version of Egorov (Exercise 1.4.31) requires $\mu(X) < \infty$, but $m(\mathbf{R}^d) = \infty$, so the previous version of Egorov (Theorem 1.3.26) isn't a special case of this one.
- ▶ Sums are now integrals with respect to the counting measure, so everything we know about integrals applies to sums (Exercise 1.4.39).

Subsection 1.4.5 is almost entirely new.