MAU 22200 Week 1 Lecture 2

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Announcements

- I initially got the permissions wrong on the module webpage, but that is now fixed. Contact me if you can't see it.
- I initially got the link wrong for Monday's lecture on Blackboard, but that is now fixed. I put this semester's lectures in a separate folder.
- There are people who couldn't do a live Q&A on Mondays or Thursdays, but no one has objected to Fridays, so I'll do them on Friday, unless someone has a conflict. This week's is on Friday in any case. I'm not sure what's the best platform, but I'll try Microsoft Teams this week.
- Assignments (exercises from the book) will start next week. I'll email you each week to let you know who is in your group and which exercise you're to do. If you don't want to use your College email for this, let me know which email you want to use.

More Announcements

- How your group wants to communicate is up to you.
- You only have one exercise per week to work on and your group is the only one working on that exercise, so try to do it correctly and write it clearly.
- Writing it up in LaTeX isn't mandatory, but it would make it much easier for the rest of the class to read. Also, learning LaTeX will be useful to you later.

Comments on Section 0.0

- The extended non-negative real axis will be very important. Make sure you understand it.
- The discussion of sums is probably unfamiliar to you, but this is a very useful way to think about them. For absolutely convergent sums the order structure on the natural numbers is irrelevant. There's a similar way to think about limits of sequences. x_n converges to y if and only if for all $\epsilon > 0$ there is a finite set $F \subset \mathbf{N}$ such that $|x_n - y| < \epsilon$ for all $n \notin F$.
- The distinction between countable and uncountable infinite sets appears for the first time in this section. It definitely won't be the last time. Make sure you understand the properties of countable sets!
- The proof of Theorem 0.0.2 is your first example of Strategy 2.1.1. (Split up equalities into inequalities).

Comments on Subsection 1.1.1

- Terry uses the word "interval" to refer to finite, but possibly empty, intervals. Not everyone uses the same conventions. Some people exclude empty intervals. Others allow semi-infinite or infinite intervals.
- An alternate characterisation of intervals is as bounded convex subsets of $\mathbf{R}$. That often helps you avoid case by case analysis. For example, it's clear that the intersection of a non-empty collections of intervals is an interval.
- Often there's more than one way to describe a set. If you're giving a definition based on a description you need to make sure that it is independent of any choices made.

More comments on Subsection 1.1.1

- As a trivial example of the last point, the length of the open interval (a, b) is defined to be $b - a$. But are a and b determined by the interval? Yes, for non-empty open intervals, but no for the empty interval. If $I = (a, b)$ and $J = (c, d)$ are non-empty then $I = J$ if and only if $a = c$ and $b = d$. But $(a, a) = \emptyset = (c, c)$ even if $a \neq c$. Length could therefore be ill-defined, but it's okay since $a - a = 0 = c - c$, so it doesn't matter which description we use.
- Often the trivial cases are the easiest to get wrong!
- For a much less trivial example of showing that a definition is independent arbitrary choices, see Lemma 1.1.2(ii).