MAU34804 2020 Practice Exam

You are allowed to use the notes originally posted to the module web page, i.e. David Wilkins' notes from last time the module was offered, your own lecture notes, the online lectures posted to the module web site after College closed and the slides that accompany them. You should not use any other resources.

Unless otherwise specified, you may use any result from any of the allowed source in any part of any problem. You may also use any result from an earlier part of a problem to do a later part, even if you didn't succeed in doing the earlier part.

Attempt all problems. All parts of all problems carry equal weight.

1. (a) We've seen that each point $\mathbf{x}$ in the polyhedron $|K|$ of a simplicial complex K in $\mathbf{R}^n$ belongs to the relative interior of exactly one simplex σ in K. Consider the correspondence S defined by $\Phi(\mathbf{x}) = \sigma$. Is Φ non-empty valued? Is it compact valued? Is it convex valued?

   Note: The questions above are questions about whether the property holds *for all* K, rather than some particular K. So a "yes" answer requires a proof valid for all K, while for a "no" answer it suffices to give a single counter-example. Also, the notes use "interior" to mean what I called "relative interior" here and in lecture.

   (b) Is Φ upper hemicontinuous? Is it lower hemicontinuous? Justify your answer.

   (c) Let Ψ be the correspondence defined by $\Psi(\mathbf{x}) = \mathrm{st}_K(\mathbf{x})$, where $\mathrm{st}_K(\mathbf{x})$ is the star neighbourhood of $\mathbf{x}$ in K, as defined in the notes and lecture. Is Ψ non-empty valued? Is it compact valued? Is it convex valued?

   (d) Is Ψ upper hemicontinuous? Is it lower hemicontinuous? Justify your answer.

2. The Prisoner's Dilemma is a two person game, but not a zero-sum two person game. The classic formulation is that of Tucker (1950):

> Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge, but they have enough to convict both on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either to betray the other by testifying that the other committed the crime, or to cooperate with the other by remaining silent. The possible outcomes are:
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> - If A and B each betray the other, each of them serves two years in prison
> - If A betrays B but B remains silent, A will be set free and B will serve three years in prison (and vice versa)
> - If A and B both remain silent, both of them will serve only one year in prison (on the lesser charge).

(a) What are the strategy sets S_A and S_B for the two "players"? Assume each player's utility is minus their expected prison sentence. What, explicitly, are the utility functions $u_A, u_B \colon S \to \mathbf{R}$, where $S = S_A \times S_B$? Do they satisfy the hypotheses of the theorem on the existence of Nash equilibria?

(b) What are the functions $b_A \colon S_B \to \mathbf{R}$ and $b_B \colon S_A \to \mathbf{R}$?

(c) What are the correspondences $B_A \colon S_B \rightrightarrows S_A$ and $B_A \colon S_B \rightrightarrows S_A$?

(d) What, if any, Nash equilibria $(\mathbf{x}_A^*; \mathbf{x}_B^*)$ does the game possess?

3. Suppose that an exchange economy has 3 goods and that household h has a linear utility function

$$u_h(x_1, x_2, x_3) = \alpha_{h1}x_1 + \alpha_{h2}x_2 + \alpha_{h3}x_3.$$

   (a) What conditions on α_{h1}, α_{h2} and α_{h3} are needed to ensure that u_h is continuous, strictly increasing and quasiconcave?

   (b) What are $B(\mathbf{p}, w)$, $V(\mathbf{p}, w)$ and $\xi(\mathbf{p}, w)$? Is V continuous? Is ξ non-empty valued, compact valued, convex valued and upper hemicontinuous?

   (c) What are $B_{\mathbf{c}}(\mathbf{p}, w)$, $V_{\mathbf{c}}(\mathbf{p}, w)$ and $\xi_{\mathbf{c}}(\mathbf{p}, w)$, where $\mathbf{c} \gg \mathbf{0}$ is arbitrary?

   (d) What is $\hat{\xi}_{\mathbf{c},h}(\mathbf{p})$, where $\mathbf{c} \gg \mathbf{0}$ and $\overline{\mathbf{x}}_h \gg \mathbf{0}$ are arbitrary?

   Note: You'll need to introduce some notation to express these in any sort of concise way. Just make sure you explain whatever notation you're using. Also, the answers will have to be expressed differently for different ranges of values. That's fine, but try to make sure you don't treat cases which can't actually occur.