MAU 34804 Lecture 30

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Questions on notes

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- *will we have to have our solutions typed to submit or will it suffice to submit scans or photos of the solutions worked out on paper?* You should get, or have got, instructions on this for all the exams. In general typing is better than scanning, if you can do it, and scanning is better than photographing.

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Final thoughts

- Good luck!
- Don't panic!