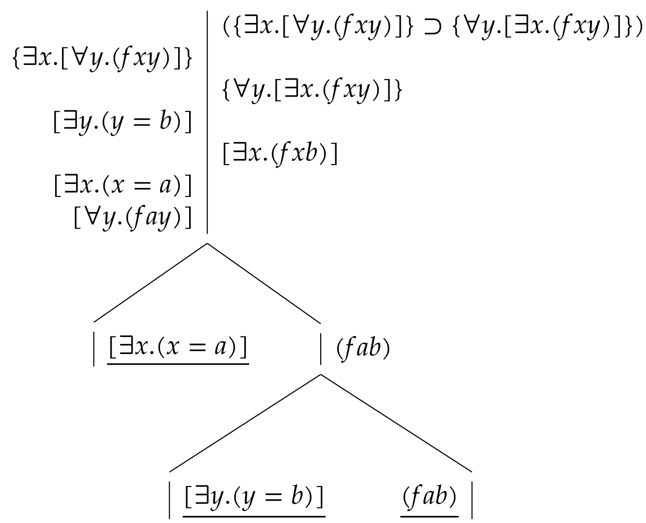
MAU11602 Assignment 5, Due Wednesday 13 March 2024
Solutions

1. Construct a tableau showing that $(\{\exists x.[\forall y.(fxy)]\} \supset \{\forall y.[\exists x.(fxy)]\})$ is valid.
Solution:



The tableau closes, so the statement is valid.

2. Translate the following informal statements about the natural numbers into the formal language from the notes.

   (a) Every number can be written as the sum of four squares.

   (b) $\sqrt{2}$ is irrational.

   (c) There are infinitely many composite numbers.

   Solution:

   (a)

$$[\forall v.(\exists w.\{\exists x.[\exists y.(\exists z.\{v = [\{[(w \cdot w) + (x \cdot x)] + (y \cdot y)\} + (z \cdot z)]\})]\})].$$

   (b) For any natural numbers x and y the number y/x is not $\sqrt{2}$, i.e.

$$\{\forall x.[\forall y.(\neg\{[(x \cdot x) + (x \cdot x)] = (y \cdot y)\})]\}.$$

   Note that there are other possibilities, like

$$\{\neg[\forall x.(\forall y.\{[[0'' \cdot (x \cdot x)] = (y \cdot y)\})]\}.$$

(c) Here we need to use the trick mentioned in the notes, that saying there are infinitely many numbers satisfying a condition is equivalent to saying that for every number there is some larger number which satisfies the condition. z is composite if and only if there are x and y both greater than 1 such that $x \cdot y = z$, i.e.

$$[\exists x.(\exists y.\{[(x > 0') \wedge (y > 0')] \wedge [(x \cdot y) = z]\})]$$

Saying that for each w there is a z greater than w which is composite is therefore

$$[\forall w.(\exists z.\{(w < z) \wedge [\exists x.(\exists y.\{[(x > 0') \wedge (y > 0')] \wedge [(x \cdot y) = z]\})]\})].$$