## MAU11602 Assignment 3, Due Wednesday 21 February 2024 Solutions

1. Use a tableau to show that Łukasiewicz' first axiom,  $[p \supset (q \supset p)]$ , is a tautology. Solution:

$$p \begin{vmatrix} p \supset (q \supset p) \\ q \supset p \end{vmatrix}$$

$$q = \begin{vmatrix} q \supset p \\ q \end{pmatrix}$$

The tableau closes, so the statement is a tautology.

2. Use a tableau to show that Łukasiewicz' second axiom,

$$\{[(\neg p) \supset (\neg q)] \supset (q \supset p)\},\$$

is a tautology. Solution:



The tableau closes, so the statement is a tautology.

3. Give a formal proof of  $\{[(\neg p) \supset (\neg q)] \supset (q \supset p)\}$  in the natural deduction system from the notes.

Solution: We introduce  $[(\neg p) \supset (\neg q)]$  as a hypothesis, use the fifth of the rules of inference given in the notes to derive  $(q \supset p)$ , and then discharge the hypothesis.

$$1 \quad [(\neg p) \supset (\neg q)]$$
  

$$2 \quad (q \supset p)$$
  

$$3 \quad \{[(\neg p) \supset (\neg q)] \supset (q \supset p)\}$$

4. Give a formal proof of Łukasiewicz' third axiom,

 $\{[p \supset (q \supset r)] \supset [(p \supset q) \supset (p \supset r)]\},\$ 

in the natural deduction system from the notes.

Solution: We introduce three hypotheses, apply modus ponens, i.e. our third rule of inference, three times, and then discharge each hypothesis in turn.