Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

## Faculty of Science, Technology, Engineering and Mathematics

## School of Mathematics

**JF Maths** **Semester 2 2025-2026**
**SF JH**

**MAU11602 Computation Theory and Logic**

**21 April 2026** **SC** **17.00–19.00**

**Prof. John Stalker**

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**Instructions to Candidates:**

Calculators or mathematical tables are permitted, but unlikely to be helpful.

**Instructions for Invigilators:**

Credit will be given for the best 3 questions answered.

1. (20 points)

   (a) (2 points) The following steps are listed in alphabetical order, but in what order should they be performed?

   - evaluation
   - lexing
   - parsing
   - type checking (or inference)

   (b) (6 points) Find the parse tree for the expression

   ```
   if 1 < 2 * 3 then 4 else 5 + 6
   ```

   (c) (6 points) Give a typing derivation to show that the expression

   ```
   if 1 < 2 * 3 then 4 else 5 + 6
   ```

   has type `bool`.

   (d) (6 points) Show all reduction steps in evaluating the expression

   ```
   if 1 < 2 * 3 then 4 else 5 + 6
   ```

2. (20 points)

(a) (8 points) Consider the following expression

$$\forall x.\forall \epsilon.\epsilon > 0 \to (\exists \delta.\delta > 0 \wedge (\forall y.\,(|x-y| < \delta) \to (|f(x)-f(y)| < \epsilon)))\,.$$

For each of the following subexpressions, which occurences of which variables are free and which are bound?

i. the whole expression ,

ii. the subexpression

$$\epsilon > 0 \to (\exists \delta.\delta > 0 \wedge (\forall y.\,(|x-y| < \delta) \to (|f(x)-f(y)| < \epsilon)))\,,$$

iii. the subexpression

$$\delta > 0 \wedge (\forall y.\,(|x-y| < \delta) \to (|f(x)-f(y)| < \epsilon))\,,$$

iv. the subexpression $|f(x)-f(y)| < \epsilon$.

(b) (4 points) Give an example of an expression and a variable which has both free and bound occurences of that variable. If this happened in the previous part you can reuse that expression. Otherwise you have to find a new example.

(c) (8 points) Perform a capture avoiding substitution of $j$ for $i$ and $2j$ for $k$ in

$$\sum_{i=j}^{k} i + \sum_{j=i}^{k} j = \frac{(j+k+1)(k-j)}{2} + \frac{(i+k+1)(k-i)}{2}$$

3. (20 points)

    (a) (4 points) What is an arithmetic function?

    (b) (4 points) Give an example of a function from the natural numbers to the natural numbers which is not arithmetic. You don't need to prove that your example works, as long as I can see that it does.

    (c) (4 points) Prove that there is a function from the natural numbers to the natural numbers which is not arithmetic. You can use your answer to the previous part, but only if you actually proved that the function you gave there is not arithmetic, using only results proved in lecture.

    (d) (4 points) Gödel's Incompleteness Theorem resulted from an ultimately unsuccessful attempt to prove what other theorem? Is that theorem now known to be true? Is it now known to be false?

    (e) (4 points) What is Rosser's Theorem, and how is it related to Gödel's Theorem?

4. (20 points)

   (a) (4 points) Let $A^c$ denote the complement of the set $A$ so that $x \in A^c$ is true if and only if $x \in A$ is false. I am just establishing notation here, not claiming that any such set exists. For which sets $A$ does $A^c$ exist?

   (b) (6 points) Justify your answer to the previous part.

   (c) (4 points) Regardless of what your answers were for the previous parts, when we establish a notation for an object whose existence we're at least temporarily unsure of we need to be careful which version of first order logic we use. Which rules of inference need to be modified to deal with this situation?

   (d) (6 points) State and prove Cantor's Theorem.