MAU11602 Assignment 8, Due Wednesday 3 April 2024

1. Suppose that S is a relation from A to B and R is a relation from B to C.

   (a) Show that if R and S are left unique then so is $R \circ S$.

   (b) Show that if R and S are right unique then so is $R \circ S$.

   (c) Show that if R and S are left total then so is $R \circ S$.

   (d) Show that if R and S are right total then so is $R \circ S$.

   (e) Show that if R and S are functions then then so is $R \circ S$.

   (f) Show that if R and S are injections then then so is $R \circ S$.

   (g) Show that if R and S are surjections then then so is $R \circ S$.

   (h) Show that if R and S are bijections then then so is $R \circ S$.

2. Show that the set of real numbers is uncountable.
   Note: We haven't formally defined the real numbers in this module, and won't, so you can use an informal definition, like the fact that each (possible infinite) decimal expansion corresponds to a real number and vice versa, if we exclude the ones ending with all 9's. You don't need to give much detail on the parts of the proofs which are real analysis rather than set theory.
   Hint: You may find it convenient to use the fact that subsets of countable sets are countable.

3. A number is called algebraic if is a root of a non-zero polynomial with rational coefficients. Using a counting argument, show that there are real numbers which are not algebraic.
   Note: You can use the result of the previous problem, even if you didn't manage to prove it.