MAU11602 Assignment 7
Due 2026-03-26

1. Consider the following two statements:

$$\forall A.\exists B.\forall x.\,((\exists C.x \in C \wedge C \in A) \rightarrow (x \in B))$$

$$\forall A.\exists B.\forall x.\,(((\exists C.x \in C \wedge C \in A) \rightarrow (x \in B)) \wedge ((x \in B) \rightarrow (\exists C.x \in C \wedge C \in A)))$$

   (a) These are two different ways to formalise one of the axioms of set theory I introduced informally in lecture. Which axiom?

   (b) It's possible to develop set theory using either of these statements as an axiom because we can get either from the other, using our other axioms and first order logic. In fact in one direction we only need first order logic. Which statement follows from the other by first order logic?

   (c) For the reverse direction we need to appeal to another axiom of set theory. Which one do we need?