MAU11602 Assignment 7, Due Wednesday 27 March 2024

1. Prove the set identity $[[A \setminus [A \setminus B]] = [A \cap B]]$.
   Hint: You can use the same method which was used in the notes for $[[[A \cup B] \cup C]] = [A \cup [B \cup C]]$:

   - Convert the equation to a pair of inclusions.
   - Write each inclusion as a substitution instance of a statement from zeroeth order logic.
   - Show that those statements are tautologies.

   In the example in the notes I skipped the last step, since I'd already given a number of examples of showing that statements are tautologies, but you shouldn't skip that step for this problem. A bit of preliminary work with the rules of inference for zeroeth order logic can simplify the statements you need to prove.

2. In the notes I defined a minimal member of a set A to be a $C \in A$ such that if $B \in A$ and $B \subseteq C$ then $B = C$ and a maximal member of A to be $B \in A$ such that if $C \in A$ and $B \subseteq C$ then $B = C$. A related pair of notions are those of a least member and a greatest member. A least member of A is a $B \in A$ such that if $C \in A$ then $B \subseteq C$. A greatest member is a $C \in A$ such that if $B \in A$ then $B \subseteq C$.

   I further defined a set E to be finite if every non-empty set of subsets of E has a minimal and a maximal member.

   (a) Give an example of a finite set E and a non-empty set A of subsets of E such that A has neither a least nor a greatest member.

   (b) Show if a set A has a least member then that member is also minimal.

   (c) Show if a set A has a greatest member then that member is also maximal.

   (d) Show that if a set A has a least member then it has no other minimal member.

   (e) Show that if a set A has a greatest member then it has no other maximal member.

   (f) Suppose E is a finite set and A is a non-empty set of subsets of A. Show that if A has only one minimal member then that member is a least member.