MAU11602 Assignment 6
Due 2026-03-19
Solutions

1. What is the modified base 5 representation of the natural number whose modifed base 7 representation is 47664?

   Solution: First we need to identify which number this is.

$$\begin{aligned}
(((4 \cdot 7 + 7) \cdot 7 + 6) \cdot 7 + 6) \cdot 7 + 4 &= (((28 + 7) \cdot 7 + 6) \cdot 7 + 6) \cdot 7 + 4 \\
&= ((35 \cdot 7 + 6) \cdot 7 + 6) \cdot 7 + 4 \\
&= ((245 + 6) \cdot 7 + 6) \cdot 7 + 4 \\
&= (251 \cdot 7 + 6) \cdot 7 + 4 \\
&= (1757 + 6) \cdot 7 + 4 \\
&= 1763 \cdot 7 + 4 \\
&= 12341 + 4 \\
&= 12345.
\end{aligned}$$

   Next we need to write this in modified base 5. 12345 is congruent to 5 modulo 5 so the last digit is a 5. $(12345 - 5)/5 = 2468$ is congruent to 3 modulo 5 so the digit before that is a 3. $(2468 - 3)/5 = 493$ is congruent to 3 modulo 5 so the digit before that is a 3. $(493-3)/5 = 98$ is congruent to 3 modulo 5 so the digit before that is a 3. $(98-3)/5 = 19$ is congruent to 4 modulo 5 so the digit before that is a 4. $(19-4)/5 = 3$ is congruent to 3 modulo 5 so the digit before that is a 3. $(3 - 3)/5$ is zero, so there are no further digits. Writing the digits we've found in the correct order we find that the modified base 5 representation of 12345 is 343335.

2. In lecture I used the fact that the composition of arithmetic functions is arithmetic, but I never actually proved it. Prove it, at last in the case of unary functions.

   Solution: Suppose f and g are arithmetic unary functions and $h = f \circ g$. In other words, there is a Boolean expression F with two free variables, y and z, which is equivalent to $z = f(y)$, i.e. the result of substituting any two natural numbers for free occurences of y and z in F is the same as substituting for them in $z = f(y)$. Similarly, there is a Boolean expression G with two free variables, x and y, which is equivalent to

$y = g(x)$. The definition of arithmetic functions just says there is such an expression with some pair of free variables, but if they aren't x and y we can always perform an α conversion to make it so they are. Now $G \to F$ is a Boolean expression with three free variables equivalent to $y = f(x) \to z = g(y)$. This is true for all y if and only if $z = f(g(x)) = h(x)$, so

$$\forall y. G \to F$$

is a Boolean expression with two free variables, x and z equivalent to $h(x) = z$ and therefore h is arithmetic.

There are lots of different possibilities for the expression equivalent to $h(x) = z$. Another one which works is

$$\exists y. F \wedge G.$$