

MAU11602 Assignment 4
Due 2026-02-26

1. (a) Write an expression with two free variables, d and n , expressing the fact that n is divisible by d , i.e. one which evaluates to true if and only if n is divisible by d . The language elements you have available are integer constants, the arithmetic operators \cdot , $+$, and $-$, the arithmetic relations $<$, \leq , $=$, \geq , and $>$, the symbols \wedge , \vee , \rightarrow , \top and \perp from zeroth order logic, and the quantifiers \forall and \exists from first order logic.
 - (b) Write an expression with a single free variable x expressing the fact that n is prime. The language elements you can use are the same as in the previous part.
2. The functions

```
fun f2 0 = 0
  | f2 1 = 1
  | f2 n = if n > 0
            then f2 (n - 2) + f2 (n - 1)
            else f2 (n + 2) - f2 (n + 1)
fun h (0, (x, y)) = x
  | h (n, (x, y)) = if n > 0
                    then h (n - 1, (y, x + y))
                    else h (n + 1, (y - x, x))
fun f4 n = h (n, (0, 1))
```

appeared in lecture as ways to compute the n 'th Fibonacci number, but I never actually proved that `f2` and `f4` compute the same value if given the same argument. Prove this. You will probably want to use induction, but in order to do this you need to find an appropriate statement to apply induction to.