MAU11602 Assignment 2, Due Wednesday 14 February 2024

1. Consider a formal system with grammar

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statement : number "+" number relation number
number : "0" | number "'"
relation : "=" | "≠"
the single axiom
0+0=0
and the following rules of inference
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- (a) From any statement containing the strings + and =0 we can deduce the same statement, but with + replaced by '+ and =0 replaced by =0'.
- (b) From any statement containing the string=0 we can deduce the same statement, but with =0 replaced by '=0'.
- (c) From any statement containing the strings + and =0 we can deduce the same statement, but with + replaced by '+ and =0 replaced by ≠0.
- (d) From any statement containing the string=0 we can deduce the same statement, but with =0 replaced by '≠0.
- (e) From any statement containing the string=0 we can deduce the same statement, but with =0 replaced by $\neq 0$ '.

Give formal proofs of the following theorems:

- (a) 0''+0'''=0''''
- (b) 0''+0''≠0''''
- 2. Every string in the language from the previous problem is a 0 followed by some number of 's followed by a + followed by a 0 followed by some number of 's followed by either a = or a \neq followed by a 0 followed by by some number of of 's. Say the numbers of 's above are *j*, *k* and *l*. A statement with a = is to be interpreted as meaning that j + k = l while one with a \neq is to be interpreted as meaning that $j + k \neq l$. Negation a statement means swapping a = for a \neq or vice versa.

With this interpretation, is the system

- (a) sound?
- (b) consistent?
- (c) semantically complete?
- (d) syntactically complete?

Note: You can't give formal proofs that any of these either are or aren't true since we don't have a formal system for describing formal systems. You can, and should, give informal proofs, but these don't have to be very detailed and you can use basic facts about arithmetic.