# MA 216
# Assignment 3
# Due 21 November 2007

1. (a) Given the Jordan decomposition

$$\begin{pmatrix} 17 & -9 \\ 25 & -13 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 5 & -3 \\ -3 & 2 \end{pmatrix},$$

solve the initial value problem

$$x'(t) = 17x(t) - 9y(t),$$

$$y'(t) = 25x(t) - 13y(t),$$

$$x(0) = \xi \qquad y(0) = \eta.$$

(b) Given the Jordan decomposition

$$\begin{pmatrix} -22 & 15 \\ -39 & 26 \end{pmatrix} = \begin{pmatrix} 2+i & 2-i \\ 3+2i & 3-2i \end{pmatrix} \begin{pmatrix} 2+3i & 0 \\ 0 & 2-3i \end{pmatrix} \begin{pmatrix} (2+3i)/2 & (-1-2i)/2 \\ (2-3i)/2 & (-1+2i)/2 \end{pmatrix},$$

solve the initial value problem

$$x'(t) = -22x(t) + 15y(t),$$

$$y'(t) = -39x(t) + 26y(t),$$

$$x(0) = \xi \qquad y(0) = \eta.$$

2. (a) Find a basis for the space of solutions to the linear homogeneous differential equation

$$x''''(t) - x(t) = 0.$$

(b) Find all solutions of the linear inhomogenous differential equation

$$x''''(t) - x(t) = t.$$

3. Solve the initial value problem for the forced harmonic oscillator

$$mx''(t) + 2\gamma x'(t) + kx(t) = C\cos(\omega t - \varphi)$$

in the exceptional case

$$\gamma = 0 \qquad \omega^2 = k/m.$$