MA 216
Assignment 2
Due 7 November 2007

1. Rewrite each of the following higher order scalar equations as a first order system.

   (a) Hook's law
   $$mx''(t) + kx(t) = 0$$

   (b) Bessel's equation
   $$t^2x''(t) + tx'(t) + (t^2 - \nu^2)x(t) = 0,$$

   (c) The Laguerre Differential Equation
   $$tx''(t) + (1 - t)x'(t) + \nu x(t) = 0.$$

2. It was shown in lecture that if $AB = BA$ then each of the following equations is satisfied for all real t:
   $$\exp(tA + tB) = \exp(tA)\exp(tB),$$
   $$\exp(tA + tB) = \exp(tB)\exp(tA),$$
   $$\exp(tA)\exp(tB) = \exp(tB)\exp(tA).$$

   Prove that, conversely, if any of these three equations is satisfied for all t then $AB = BA$.
   Hint: Differentiate twice.

3. Compute the following matrix exponentials:

   (a)

$$\exp\left(t\begin{pmatrix} 13 & -5 \\ 30 & -12 \end{pmatrix}\right)$$

   (b)

$$\exp\left(t\begin{pmatrix} 1 & 1 \\ -4 & 5 \end{pmatrix}\right)$$

   (c)

$$\exp\left(t\begin{pmatrix} -2 & 2 \\ -13 & 8 \end{pmatrix}\right)$$

4. Find the solution to the initial value problem

$$x(0) = \xi_1 \qquad x'(0) = \xi_2$$

   for the general linear constant coefficient second order scalar equation

$$c_2 x''(t) + c_1 x'(t) + c_0 x(t) = 0$$

   by reducing it to a first order system.