MA 419 Assignment 3

Due 17.00 on 8 December 2006 in the Maths Office

1. Solve the initial value problem

$$u(0, x) = x^2 \qquad u_t(0, x) = 0$$

for the wave equation

$$u_{tt} - c^2 u_{xx} = 0$$

on the half line $x \geq 0$ with Dirichlet boundary conditions

$$u(t, 0) = 0$$

by the method of reflection.

2. Solve the initial value problem

$$u|_{t=0} = \varphi$$

for the Diffusion equation

$$u_t - k u_{xx} = 0$$

on the interval $[0, l]$ with Neumann boundary conditions at the two ends

$$u_x|_{x=0} = 0 = u_x|_{x=l}$$

by the method of reflection.

3. Solve the same problem by the method of separation of variables. You may omit the hard step, *i.e.* showing that any reasonable initial data φ can be represented as an infinite sum of the appropriate type, but you should calculate the coefficients and state clearly what result you would need to complete the proof.

4. Prove conservation of energy in the form

$$\frac{d}{dt} \int_0^\infty \left(u_t^2 + c^2 u_x^2 \right) dx = 0$$

for solutions of the wave equation

$$u_{tt} - c^2 u_{xx} = 0$$

with either Dirichlet or Neuman boundary conditions. Does the result hold for the Robin condition?