MA 419 Assignment 2

Due 15 November 2006

1. Compute $f \star g$ where $f$ and $g$ are defined by

$$f(x) = \exp(-|x|)$$

and

$$g(x) = \begin{cases} 0 & \text{if } x < -1 \\ 1 & \text{if } -1 \leq x \leq 1 \\ 0 & \text{if } 1 < x \end{cases} .$$

2. The solution of the initial value problem for the homogeneous Wave Equation

$$u_{tt} - c^2 u_{xx} = 0 \qquad u|_{t=0} = \varphi \qquad u_t|_{t=0} = \psi$$

in the special case $\varphi = 0$ can be written in the form

$$u(t, \cdot) = K(t, \cdot) \star \psi$$

for an appropriate function $K$. What is this $K$?

3. A function $f$ is said to be $\alpha$-Hölder continuous if, for every $x$, there is an $L$ and a $\delta > 0$ such that

$$|f(y) - f(x)| < L|y - x|^{\alpha}$$

whenever $|y - x| < \delta$. Show that if there is a positive $\alpha$ such that $f$ is $\alpha$-Hölder continuous then $f$ is continuous. Show that if $f' \in L^p(\mathbf{R})$ for some $1 \leq p \leq \infty$ then $f$ is $\alpha$-Hölder continuous for

$$\alpha = 1 - \frac{1}{p}.$$

4. Solve the Diffusion Equation

$$u_t - ku_{xx}$$

with initial data

$$u(0, x) = e^{\lambda x}.$$

5. With $S$ defined, as in lecture, by

$$S(t, x) = (4\pi kt)^{-1/2} \exp(-\frac{x^2}{4kt}),$$

prove that for all positive $s$ and $t$

$$S(s + t, \cdot) = S(s, \cdot) \star S(t, \cdot).$$