MA 419 Assignment 1

Due Wednesday 25 October 2006

1. For each of the following, say whether it is a scalar equation or system, give the order, and state whether it is linear or non-linear. If linear, state whether it is homogeneous or inhomogenous.

   (a) The 1+3 Dimensional Wave Equation

   $$u_{tt} - u_{xx} - u_{yy} - u_{zz} = 0$$

   (b) The 1+1 Dimensional Klein-Gordon Equation

   $$u_{tt} - u_{xx} = u$$

   (c) The 1+3 Dimensional Eikonal Equation

   $$u_t^2 - u_x^2 - u_y^2 - u_z^2$$

   (d) The Incompressible Euler Equations

   $$\begin{aligned} u_t + uu_x + vu_y + wu_z + p_x &= 0 \\ v_t + uv_x + vv_y + wv_z + p_y &= 0 \\ w_t + uw_x + vw_y + ww_z + p_z &= 0 \\ u_x + v_y + w_z &= 0 \end{aligned}$$

   (e) The Euler-Tricomi Equation

   $$u_{xx} = xu_{yy}$$

   (f) The Korteweg-de Vries Equation

   $$u_t + u_{xxx} - 6uu_x = 0$$

2. Find all solutions of the first order linear homogeneous equation

   $$u_t - xu_x = 0$$

3. Find all solutions of the first order linear homogeneous equation

   $$u_t - xu_x + tu = 0$$

4. Solve the initial value problem for the Wave Equation

   $$u_{tt} - u_{xx} = 0$$

   with initial data

   $$u(0, x) = 0 \qquad u_t(0, x) = \frac{1}{1 + x^2}.$$