MA 216 Assignment 4

Due 7 February 2007

1. The Laguerre Differential Equation is

$$tx''(t) + (1-t)x'(t) + \nu x(t) = 0.$$

For which ν does this equation have a polynomial solution? For these ν, what is this solution?

2. Suppose g is a continuously differentiable function with $g(0) \neq 0$. 0 is an equilibrium of

$$x'(t) = x(t)g(x(t)).$$

Under what conditions on g is this equilibrium stable? Under what conditions is it strictly stable?

3. Find the equilibria of the system

$$x'(t) = x(t)^2 + 3y(t)^2 - 4,$$

$$y'(t) = 8x(t)^2 + y(t)^2 - 9.$$

Which are stable? Which are strictly stable?

4. Prove that

$$V(x,y) = 5x^2 + 8xy + 5y^2$$

is a Lyapunov function, but not a strict Lyapunov function for the system

$$x' = -15x^3 - 24x^2y - 15xy^2 - 4x - 5y,$$

$$y' = -15x^2y - 24xy^2 - 15y^3 + 5x + 4y.$$