MA 216 Assignment 3

Due 17.00, 8 December 2006 in the Maths Office

1. Find all solutions of the differential equation

$$x'(t) + tx(t) + t^2 = 0.$$

2. Solve the initial value problem

$$\begin{aligned} x'(t) &= x(t) + \exp(t)y(t), \\ y'(t) &= \qquad\qquad -y(t), \end{aligned}$$

$$\begin{aligned} x(0) &= 0, \\ y(0) &= 1. \end{aligned}$$

3. A particular solution to the differential equation

$$(t^4 + t^2 + 4)x''(t) - (4t^3 + 2t)x'(t) + (6t^2 + 8)x(t) = 0$$

is

$$x_1(t) = t^3 - 4t.$$

Find the general solution.

4. The differential equation

$$x''(t) + (2n + 1 - t^2)x(t) = 0,$$

which appears in the study of the harmonic oscillator in Quantum Mechanics, has a solution of the form

$$x(t) = H_n(t)\exp(-t^2/2),$$

where H_n is a polynomial of degree n. These are called the Hermite Polynomials, but neither the name nor their exact form is relevant to this problem. Show that

$$\int_{-\infty}^{+\infty} H_m(t)H_n(t)\exp(-t^2)\,dt = 0$$

if $m \neq n$.