MA 216 Assignment 1

Due Wednesday 25 October 2006

1. For each of the following, say whether it is a scalar equation or system, give the order, and state whether it is linear or non-linear. If linear, state whether it is homogeneous or inhomogenous.

   (a) Van der Pol's Equation:

   $$x''(t) + (1 - x(t)^2)x'(t) + x(t) = 0$$

   (b) Bessel's Equation:

   $$t^2x''(t) + tx'(t) + (t^2 - \nu^2)x(t) = 0$$

   (c) The Emden-Fowler Equation

   $$tx''(t) + 2x'(t) + at^\nu x(t)^n = 0.$$

   (d) The Lotka-Volterra Model:

   $$x'(t) = x(t)(\alpha - \beta y(t)) \qquad y'(t) = -y(t)(\gamma - \delta x(t))$$

2. Which of the equations (or systems) from the preceding exercise have translation symmetry in the independent variable (t as the equations are written above)?

3. (a) Prove that

   $$z(t) = x(t)y(t)$$

   is an invariant of the system

   $$x'(t) = x(t) \qquad y'(t) = -y(t).$$

   Note: although the system is easy to solve, there is no need to do so.

   (b) Prove that

   $$x'(t)^2 + x(t)^4$$

   is an invariant of

   $$x''(t) + 2x(t)^3 = 0$$

   and use this fact to prove that all solutions of the equation are bounded.

4. Prove that

$$\|AB\|_\infty \leq \|A\|_\infty \|B\|_\infty.$$

5. A vector valued function x is said to be continuous at t if, for all positive ϵ, there is a positive δ, such that

$$|s - t| < \delta$$

implies

$$\|x(s) - x(t)\| < \epsilon.$$

Show that it doesn't matter which norm we take, $\|x\|_1$ or $\|x\|_\infty$, in this definition, *i.e.* that the same set of functions are continuous regardless of which norm is used.