

## Assignment 6

### 443 Statistical Physics 2008/2009

Lecturer: Stefan Sint

(due Wednesday, 1 April 2009 during class)

#### Problem 1 (8/40 points)

A classical gas of  $N$  particles is confined to a region  $R$  of volume  $V$  and described by the canonical distribution function

$$f(p, q) = \begin{cases} Z^{-1} \exp\left(-\beta \sum_{i=1}^N \frac{\mathbf{p}_i^2}{2m}\right), & \text{if } \mathbf{q}_i \in R \text{ for all } i = 1, \dots, N \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

- a) Show that the probability distribution for the  $z$ -component  $\mathbf{v}_{i,z}$  of the  $i$ th particle's velocity is given by Maxwell's distribution,

$$p(v_z)dv_z = C e^{-mv_z^2/(2kT)} dv_z$$

- b) Determine the normalisation constant  $C$  and calculate the expectation values

$$\langle \mathbf{v}_i \rangle, \quad \langle v_i \rangle, \quad \langle (\mathbf{v}_i - \langle \mathbf{v}_i \rangle)^2 \rangle,$$

where  $v_i$  denotes the modulus of  $\mathbf{v}_i$ .

- c) Which is the most probable value of  $v_i$ ?

#### Problem 2 (8/40 points)

A non-interacting, non-relativistic gas of atoms of mass  $m$  is enclosed in a box of volume  $V$  and each of the atoms is initially in an excited state. When falling back to their ground state, they emit light which is observed in the  $z$ -direction using a spectrometer. An atom at rest would emit light corresponding to the (known) energy  $E_0$ . However, due to thermal motion of the atoms and the Doppler effect, the spectrometer records a distribution of energy values  $E$  with an intensity distribution  $I(E)$ .

- a) Compute the average of the observed energies  $\langle E \rangle$ .

- b) Compute the variance  $\langle (E - \langle E \rangle)^2 \rangle$ .
- c) Compute the intensity distribution  $I(E)$ , defined as the probability density for observing the energy  $E$ .

**Problem 3 (8/40 points)**

A classical ideal gas of  $N$  atoms is contained in a cubic box of volume  $V = L_x L_y L_z$ . In the middle of one of the walls there is a very little hole with area  $A$ , and it is assumed that there is a vacuum outside the box.

- a) What is the rate of atoms leaving the container?
- b) How long does it take for the pressure to decrease by a factor  $1/e$ ?

**Problem 4 (8/40 points)**

Consider a *non*-degenerate ideal Bose gas (i.e.  $z \ll 1$ ), with non-relativistic one-particle energies  $\mathbf{p}^2/(2m)$  and fixed average particle number  $\bar{N}$ . Derive the virial expansion

$$\frac{PV}{\bar{N}KT} = a_1 + a_2 t + a_3 t^2 + \dots, \quad t = \frac{\bar{N} \lambda^3}{n_{pol} V}$$

and compute the virial coefficients  $a_1$ ,  $a_2$  and  $a_3$ . Here,  $\lambda = h/\sqrt{2\pi m k T}$  denotes the thermal wavelength and  $n_{pol}$  is the number of polarisation states.

**Problem 5 (8/40 points)**

Consider a classical one-dimensional harmonic oscillator with the Hamilton function

$$\mathcal{H}(p, q) = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 q^2$$

Compute

- a) the phase space volume

$$\varphi(E) = \int dp dq \theta(E - \mathcal{H}),$$

- b) the contents of the energy shell,

$$\Omega(E) = \int dp dq \delta(\mathcal{H} - E).$$

Here  $\theta$  denotes the Heavyside step function and  $\delta$  is the Dirac  $\delta$ -function.