

## Assignment 4

443 Statistical Physics 2008/2009

Lecturer: Stefan Sint

(due Friday, 9 January 2009 during class)

### Problem 1 (12/20 points)

A non-ideal die gives an average result of 4 rather than  $7/2$ .

- Determine the probabilities  $p_i$ , ( $i = 1, \dots, 6$ ) if no further information is given, by maximising the information entropy with the given constraint. Use Lagrange multipliers as we did in class, and give an approximate numerical solution to the resulting transcendental equation.
- Calculate the information entropy for this solution.
- Find 2 further probability distributions which give the same average of 4 and calculate their information entropy.

### Problem 2 (8/20 points)

Consider the probability density  $\rho$  of a single real variable  $x$ . Expectation values are defined by

$$\langle f \rangle = \int_{-\infty}^{\infty} dx f(x) \rho(x).$$

We assume that the probability density is normalised and centered at  $x = 0$ , i.e.

$$\langle 1 \rangle = \int_{-\infty}^{\infty} dx \rho(x) = 1, \quad \langle x \rangle = \int_{-\infty}^{\infty} dx x \rho(x) = 0.$$

Maximise the information entropy to determine the most likely distribution with the constraint

$$\langle x^2 \rangle = \sigma^2,$$

where  $\sigma^2$  is a given positive constant.