Course 345: INTRODUCTION TO SOLITONS

Problem Set 1

Date Issued: January 28, 2008
Date due: February 4, 2008

Each problem counts 5 points

1. Obtain the solution of the equation

$$u_t + (1+u)u_x = 0$$

with
(a) $u(x,0) = u_0 x, \quad 0 \leq x \leq 1$; (b) $u(x,0) = u_0(2-x), \quad 1 \leq x \leq 2$;
(c) $u(x,0) = 0, \quad x < 0, x \geq 2$.

2. Transform the KdV equation $u_t - 6uu_x + u_{xxx} = 0$ by substituting

$$u(x,t) = -(3t)^{-2/3} f(\eta); \qquad \eta = x/(3t)^{1/3}$$

3. Show that

$$u(x,t) = \frac{6x(x^3 - 24t)}{(x^3 + 12t^2)^2}$$

is a rational solution of the KdV equation $u_t - 6uu_x + u_{xxx} = 0$.

4. Find solitary-wave solution of the Boussinesq equation

$$u_{tt} - u_{xx} + 3(u^2)_{xx} - u_{xxxx} = 0$$

in the form $u(x,t) = a\,\mathrm{sech}^2[b(x - \omega t)]$ where a, b, ω are constants. Show that a wave may propagate in either direction.

5. Find a solitary-wave solution of a modified KdV equation

$$u_t + 6u^2 u_x + u_{xxx} = 0$$

with $u, u_x, u_{xx} \to 0$ as $|x| \to \infty$.

6. Show that the modified KdV equation above has the rational solution

$$u(x,t) = c - \frac{4c}{4c^2(x - 6c^2 t)^2 + 1}$$