Course 141: MECHANICS

Problem Set 7

Date Issued: December 5, 2007
Date due: January 14, 2008

Bonus Set: Each problem counts 15 points

1. The tides:

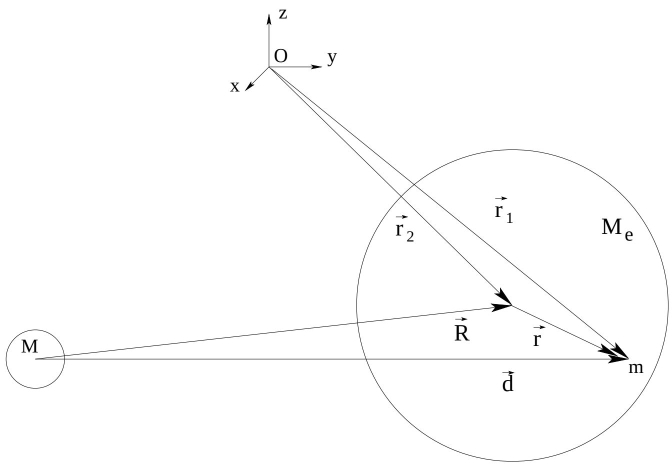
   (a) Assume earth is entirely covered by water. The ocean tides are due to the gravitational attraction of the moon and sun on the ocean surface. To see this, consider a mass m on the ocean's surface and a distant mass M, as shown. It is not necessary to assume that M is a point mass if M is spherically symmetric. Let M_e is the earth's mass. Write down the forces acting on m and M_e. Show that (see figure)

$$\ddot{\vec{r}} = -\frac{GM_e}{r^2}\hat{r} - GM\left(\frac{\hat{d}}{d^2} - \frac{\hat{R}}{r^2}\right)$$

   The first term is simply the earth's gravitational force per unit mass; the second is the tide generating force per unit mass, i.e., a mass m of water on the ocean's surface experiences a tidal force

$$\vec{F} = -mM\left(\frac{\hat{d}}{d^2} - \frac{\hat{R}}{R^2}\right)$$

   Perform the calculations in the inertial frame O not associated with earth.



   (b) Assuming M is above the equator, make a sketch of the magnitude and direction of the tidal forces on the ocean surface, North, South, East and in between.

2. A particle is dropped from a 100 m tower on the equator. Where does the particle land? Do calculations in inertial frame. (Answer: 2.2 cm Eastwards from tower's base)

3. Assume that the sun's density as a function of distance r from its centre is described by

$$\rho(r) = \rho_c[1 - (r/R)^\alpha],$$

where R is sun's radius, ρ_c sun's central density, and α is to be determinated.

   (a) Find the sun's mass as a function of r, i.e., calculate

$$m(r) = 4\pi \int_0^r \rho(r')r'^2 dr'$$

   (b) Express ρ_c in term of the sun's total mass M, raduiu R and α. It is known that $M = 1.99 \cdot 10^{30}\ kg$, $R = 6.96 \cdot 10^8\ m$. Calculate the average density of the sun, ρ_{av}, in kg/m^3. Express ρ_c in terms of ρ_{av} and α.

   The sun is transforming hydrogen into helium in its core, and so the density there must be high. This requires $\alpha < 1$. Suppose $\alpha = 1$. What is the density ar $r = R/2$ in terms of ρ_{av}? Repeat this calculation with $\alpha = 0.1$ and $\alpha = 0.01$. You will see that α is a rapidly changing function of $\rho(R/2)$. Set $\alpha = 0.1$, then the density at $r = R/2$ is a little more than twice the sun's average density, while at $r = 0$ it is about 30 times ρ_{av}. For $\alpha = 0.1$ you should find $\rho_c = 4.3{\cdot}10^4\ kg/m^3$. Tne observed value is $1.6 \cdot 10^5\ kg/m^3$, so the choice $\alpha \sim 0.1$ is not bad! Make a sketch of $\rho(r)$ versus r.

   (c) Find a general formula for the pressure at the sun's centre using

$$\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

   Express your answer in terms of G, M, R and α. Check that your answer is dimensionally correcr, i.e., force/area. Calculate the central pressure using the value of α in (b). (Hint: Obviously, the pressure on the sun's surface, $R(R) = 0$.)

   (d) Assuming the perfect gas law holds, i.e., $PV = NKT$ at the sun's centre (here N is a number of particels in volume V), and the sun is mainly hydrogen, estimate the temperature of the sun's core. Assume all hydrogen at the sun's core is ionized. Compare your answer with the observed value $T \sim 1.5 \cdot 10^7 K^\circ$.

   Data: $k = 1.38 \cdot 10^-23\ N\ m/K$ (Boltzmann's constant), proton mass $\sim 1.67 \cdot 10^{-27}\ kg$.