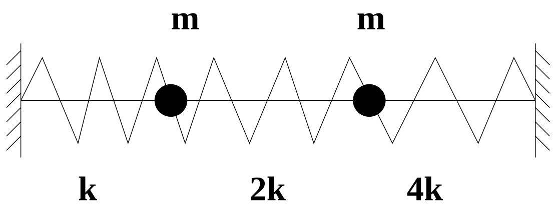
Course 141: MECHANICS

Problem Set 20

Date Issued: May 7, 2008

1. Two particles of mass m are connected between three springs and perform longitudinal oscillations. The strengths of the three springs are k. $2k$ and $4k$, respectively. Deduce the equations of motion and find the normal modes of the oscillation. Is the general motion periodic?



2. The wave on the string propagates with a displacement

$$y = A\sin(\omega t - kx)$$

Show that the average rate of working by the force (average value of transverse force times transverse velocity) equals the rate of energy transfer along the string.

3. A particle of mass m is situated on the axis of a uniform disk, of mass M and radius R. Find the gravitational force that the disk exerts on the particle.

4. The trajectory of a charged particle moving in a magnetic field is given by

$$\vec{r} = a\cos\omega t\hat{\mathbf{i}} + a\sin\omega t\hat{\mathbf{j}} + ct\hat{\mathbf{k}}$$

where a, ω and c are some constants. Show that particle moves with constant speed and find the magnitude of its acceleration.

5. A particle is sliding along a smooth radial grove in a circular turntable which is rotating with constant angular velocity Ω. The distance of the particle from the rotation axis at time t is observed to be $r = b\cosh\Omega t$ where b is a positive constant. Find the speed of the particle (relative to a fixed reference frame) and find the magnitude and direction of the acceleration.

6. A body is projected vertically upwards with speed v_0 in a medium that exerts a drag force $-mk\vec{v}$ where k is a positive constant and $\vec{v}$ is the velocity of the body. Find the maximum height achieved by the body, the time taken to reach that height, and the terminal speed.