Course 141: MECHANICS

Problem Set 1

Date Issued: October 31, 2007

1. Given $\vec{a} = 3\hat{\imath} + \hat{\jmath}$, $\vec{b} = -\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$, $\vec{c} = 7\hat{\imath} - \hat{k}$ (in meters).

   (a) Calculate $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$, $\vec{a} \times (\vec{b} \times \vec{c})$

   (b) Verify that $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}\,(\vec{a} \cdot \vec{c}) - \vec{c}\,(\vec{a} \cdot \vec{b})$.

2. A particle position at time t (in sec.) is given by

$$\vec{r}(t) = \hat{\imath}t + \hat{\jmath}\sin t + \hat{k}\cos t \qquad \text{(in meters).}$$

   (a) Sketch the particle's trajectory.

   (b) Sketch the particle's velocity $\vec{v}$.

   (c) What is the particle's speed $v = |\vec{v}|$?

   (d) How far does the particle travel between $t = 0$ and $t = 2$ *sec.* along its trajectory?

   (e) What is the particle's acceleration $\vec{a}$ at $t = 0$?

3. A particle is moving such that its position vector at time t is $\vec{r}_1 = (6t^3 - 4t)\hat{\imath} + (t^2 - 2t)\hat{\jmath} + t^3\hat{k}$ meters, a second particle is moving such taht its position vector is $\vec{r}_2 = t^2\hat{\imath} - (8t^3 + 6t)\hat{\jmath} + t\hat{k}$ meters.

   (a) Obtain the relative position vector $\vec{r}$, of particle 2 with respect to particle 1. Write down the relative velocity $\vec{v}$ and relative acceleration $\vec{a}$ at tine t.

   (b) Find the component of relative velocity, $\vec{v}$ along the direction $c = 4\hat{\imath} - 3\hat{\jmath}$ at $t = 2$ seconds.

4. Recall: The amount of work, dW, done by a force $\vec{F}$ on a particle whose change in position is $d\vec{r}$ is defined as (in Joules)

$$dW = \vec{F} \cdot d\vec{r}$$

   Suppose $\vec{F} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$ Newtons, the particle's initial position is $\vec{r}_1 = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ meters, and its final position is $\vec{r}_2 = \hat{\imath} - \hat{k}$ meters. Find the work done by $\vec{F}$.

5. For the force $\vec{F} = y^2\hat{\imath} + \hat{\jmath}$ calculate the work done over the two path I and II shown below.