Course 141: MECHANICS

Problem Set 19

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1. The spherical pendulum. A particle of mass M is suspended from a fixed point O by a light inextensible string of length l and moves in with the string taut in three-dimensional space. Initially the string makes an angle α with the downward vertical and the particle is projected with speed v in a horizontal direction at right angle to the string.
   (a) Show that angular momentum about the vertical axis through the point O is conserved and express this conservation law in terms of the coordinates θ (polar angle) and ϕ (azimuthal angle).
   (b) Obtain the corresponding equation for conservation of energy.
   (c) Determine the constants of motion and deduce an equation satisfied by $\theta(t)$ in the motion.
   (d) How does the motion actually look like? Is it planar?

2. A particle slides on the smooth inner surface of a circular cone with semi-angle α. The axis of symmetry of the cone is vertical with the vertex O pointing downwards.
   (a) Show that the vertical component of angular momentum about O is conserved in the motion. State a second dynamical quantity that is conserved.
   (b) Initially particle is a distance a from O when it is projected horizontally along the inside surface of the cone with speed v. Show that, in the subsequent motion, the distance r from O satisfies the equation

$$\dot{r}^2 = (r-a)\left[\frac{v^2(r+a)}{r^2} - 2g\cos\alpha\right]$$

   (c) For the case when gravity is absent, find r and the azimuthal angle ϕ explicitly as functions of t. Make a sketch of the path as seen from above, when $\alpha = \pi/6$.

3. A circular disk of mass M and radius r is smoothly pivoted about its axis of symmetry which is fixed in a horizontal position. A bug of muss m runs with constant speed v around the rim of the disk. Initially the disk is held at rest and is released when the bug reaches its lowest point. What is the condition that the bug will reach the highest point of the disk?

4. Two pendulums, each a light rod l supporting masses m_1 and m_2, respectively, are coupled by a weightless string of stiffness k. Write corresponding equations of motion and find the normal coordinates and normal modes for that system.

5. The displacement of a wave on a string which is fixed at both ends is given by

$$y(x,t) = A\cos(\omega t - kx) + \kappa A\cos(\omega t + kx)$$

   where κ is the coefficient of amplitude reflection. Show that this may be expressed as the superposition of the standing waves

$$y(x,t) = A(1+\kappa)\cos\omega t\cos kx + A(1-\kappa)\sin\omega t\sin kx$$