Course 141: MECHANICS

Problem Set 13

Date Issued: February 20, 2008
Date due: February 27, 2008

1. (5 points) Discuss the possible types of orbits for a particle moving under a central inverse-cube law force, decribed by the potential energy function $V = \frac{k}{2r^2}$. For the repulsive case ($k > 0$), show that the orbit equation is

$$r \cos n \; (\theta - \theta_0) = b$$

where n, b and θ are constants. Show that for the attractive case the nature of the orbit depends on the signs of $J^2 = mk$ and E. Find the equation of the orbit for each possible type. (Include the cases where one of these parameters vanishes.)

2. (5 points) A particle of mass m moves under the influence of the force $\vec{F} = -\frac{c\vec{r}}{r^{5/2}}$
   (a) Calculate the potential energy.
   (b) By means of the effective potential energy and the energy diagramm discuss the motion.
   (c) Find the radius of any circular orbit in terms of the angular momentum and calculate the period for the orbit.

3. (5 points) Two equal point masses m are connected by a string which passes through a small hole on a frictionless mass on the table for vertical up-and-down motion of the suspended mass.
   (a) Obtain the radial equation and find its solution for circular orbits.
   (b) Evaluate the minimum of the effective potential energy. By means of the energy diagramm define the allowed physical region for motion and discuss the motion.
   (c) Consider small radial perturbations of the circular orbit of the form $r(t) = r_0 + \delta(t)$ where $\delta(t) \ll r_0$. Describe the effect of this small radial impulsive blow on the orbital motion.

4. (5 points) A planet moves in a circular orbit about a massive star with force law given by $\vec{F} = -\dfrac{\alpha\vec{r}}{r^3}$. The star evolves into a supernova and blows off half its mass in a time short compared to the planet's orbital period. Assume that the supernova explosion is spherically symmetric. Show that the planet's orbit becomes parabolic.

5. (5 points) Consider the motion of a particle in the central force $\vec{F} = -k\vec{r}$. Show that
   (a) The orbit is an ellipse with the force center at the center of the ellipse.
   (b) The period is independent of the orbit parameters. Hint: use cartesian coordinates to solve for the orbit equation.