Course 141: MECHANICS

Problem Set 11

Date Issued: February 6, 2008
Date due: February 13, 2008

1. (5 points) A 0.3 kg mass is attached to a spring and oscillates at 2 Hz with a quality factor $Q = 60$. Fing the spring constant k and damping constant γ.

2. (5 points) Solve the damped unforced oscillator by the following method. Define a new variable y by
$$x = e^{\beta t} y,$$
where β is a constant. Subsitute into the equation of the damped unforced oscillator to find the equation satisfied by $y(t)$. Choose β such that the coefficient of $\dot{y}$ vanises and solve in the cases of weak, strong and critical damping.

3. (5 points) Write down the solution to the unforced oscillator equation for the case $\omega_0 > \gamma$ if the oscillator starts from $x = 0$ with velocity v. Show that as ω_0 is reduced to the critical value $\omega_0 = \gamma$, the solution tends to the corresponding solution for the critically damped oscillator.

4. (5 points) Find the steady-state solution for a damped harmonic oscillator driven by the force $F(t) = mF_0 \sin(\omega t)$.

5. (10 points) A particle of mass m moves under the action of a force
$$F = -F_0 \sinh(ax) = -\frac{F_0}{2}\left(e^{ax} - e^{-ax}\right)$$
where $a > 0$. Sketch the potential energy, discuss the motion, and solve for the frequency of small oscillations if there exists a point of stability.

6. (10 points bonus problem) A particle moving under a conservative force oscillates between x_1 and x_2. Show that the period of oscillations is
$$T = 2\int_{x_1}^{x_2} dx \left(\frac{m}{2[V(x_2) - V(x_1)]}\right)^{1/2}$$
In particular, if $V = m\omega_0^2(x^2 - bx^4)/2$, show that the period of oscillations of amplitude a is
$$T = \frac{2}{\omega_0}\int_{-a}^{a} \frac{dx}{(a^2 - x^2)^{1/2}[1 - b(a^2 + x^2)]^{1/2}}$$