Brownian Motion: From Pollen Grains to the Stock Market.

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1827-Robert Brown, botanist, noticed the jittering motion of pollen grains suspended in water.

Jittering movement was observed in both inorganic and organic particles \(\Rightarrow\) not life related.

Brownian Motion was unexplainable using physics of the day.

http://tinyurl.com/o6loj9t
1905-Einstein explained the precise motion of Brown’s particles.

The random motion of the pollen grains was due to their collisions with the water molecules.

Explanation confirmed that atoms and molecules exist.

http://tinyurl.com/nna2ceg
The diffusion equation

\[ \frac{\partial \rho}{\partial t} = D \frac{\partial^2 \rho}{\partial x^2} \]  

(1)

where \( \rho(x, t) \) = “Density of Brownian particles” and \( D = \text{“Diffusion Constant”} \) with solution:

\[ \rho(x, t) = \frac{\rho_0}{\sqrt{4\piDt}} \exp \left( \frac{-x^2}{4Dt} \right) \]  

(2)

As \((x, t) \to (\infty, \infty) \Rightarrow \rho(x, t) \to 0\)
A Stochastic Process is a collection of random variables that characterise the evolution of a random value over time.

Stochastic Process = Time & Randomness

\[ d(\text{variable}) = \lambda_1 \times d(\text{time}) + \lambda_2 \times d(\text{randomness}) \]

**Example-Weather**

The temperature in Rio de Janeiro is partly deterministic (lower temperatures are expected at night and in the winter), and partly random. In 1997 the hottest day in Rio de Janeiro occurred in the winter. The same happened in 1999!
Random Walk: A random walk with a small step size is an approximation to Brownian Motion.
Geometric Brownian Motion

It’s a continuous-time stochastic process where logarithm of random varying quantity follows a Brownian Motion with a Drift.

\[
\frac{dS_t}{S_t} = \mu dt + \sigma dW_t \quad (3)
\]

\[
S_t = S_0 \exp \left( \left( \mu - \frac{\sigma^2}{2} \right) t + \sigma W_t \right) \quad (4)
\]

A Stochastic Process is said to follow a GBM if it satisfies:
1900-Louis Bachelier-PhD-The theory of Speculation. Brownian Motion to evaluate Stock Options.

Historically first paper to use advanced mathematics in study of finance.
- 1959-M.F.M Osborne-logarithms of common stock prices, varying with time-close analogy with the ensemble of coordinates of a large number of molecules. 
  \[ \implies \text{GBM in Action.} \]
Thank You!
A. Einstein.

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