HMI Workshop on CR Geometry

All talks are in the Salmon lecture theater of the Hamilton building

(marked “Hamilton Conference Centre” on the map)

Schedule:

	Mon 24th	Tues 25th	Wed 26th
9:30 – 10:30	Shaw	Dinh	Lamel
10:30 – 11	*Coffee*	*Coffee*	*Coffee*
11 – 12	Dall'Ara	Herrmann	Zaitsev
12 – 1	Gupta	Hsiao	
	Lunch	*Lunch*	
2:30 – 3:30	Fornaess	Aidoo	
3:30 – 4	*Reception*	*Coffee*	
4 – 5		Nicoara	
6 – midnight		*Dinner at J.R. Mahon’s*	

Titles and Abstracts:

1. Nicholas Aidoo (TCD)

Normalization of Boundary Systems for Sum of Squares Hypersurfaces

Catlin introduced the multitype $M(z)$ and the commutator multitype $C(z)$ in his study of the boundary regularity for the $\bar{\partial}$-Neumann problem, and proved that $M(z) = C(z)$. While one direction of this equality is relatively direct, the reverse inequality relies on a technically involved simultaneous inductive argument.

Basyrov, Nicoara, and Zaitsev showed that the polynomial model of any pseudoconvex hypersurface of finite Catlin multitype contains a balanced sum of squares. In general, however, the associated boundary system need not admit a complete normalization due to the possible occurrence of torsion. In this talk, I consider sum of squares hypersurfaces defined by

$$r(z) = 2\mathrm{Re}\, z_1 + \sum_{j=1}^{N} |z_2, \ldots, z_n|^2.$$

I will show that for sum of squares hypersurfaces of finite Catlin multitype, the additional structure of the defining function allows the Catlin boundary system to be completely normalized. As an application, this normalization yields a direct proof of $C(z) \leq M(z)$, and hence of $C(z) = M(z)$, in the sum-of-squares setting.

2. Gautam Bharali (IISc) cancelled

Regular extensions of proper holomorphic maps: the unequal dimensions case

Abstract: We shall explore the problem of continuous extension up to $\overline{D}$ of a proper holomorphic map $F : D \to \Omega$ between domains in complex Euclidean spaces of differing dimension. In contrast to the equidimensional case, a proper holomorphic mapping $F : D \to \Omega$ does not, in general, extend even as a continuous mapping of $\overline{D}$ when the dimensions of D and Ω differ, which is a phenomenon that I will survey. We shall then look at a positive result (in the non-equidimensional case) that applies to a wide range of (pairs of) domains that includes all pairs (D, Ω) of bounded strongly pseudoconvex domains. Somewhat unexpectedly, our proofs involve solving an auxiliary complex Monge–-Ampere equation. To the best of my knowledge, these results are the first application outside the realm of PDEs of the regularity theory of the Dirichlet problem for the complex Monge–Ampere equation on domains in $\mathbb{C}^n$ (n > 1) that are not balls. This talk is based on a joint paper with Rumpa Masanta.

3. Gian Maria Dall'Ara (Pisa)

Failure of exact regularity for worm-like sub-Laplacians

Abstract: In an important 1996 paper, M. Christ proved that the $\overline{\partial}$-Neumann problem fails to be globally regular on Diederich--Fornaess worm domains. One reason this result is remarkable is that it posed a serious obstacle to proving, via $\overline{\partial}$-Neumann methods, the smooth extendability of biholomorphisms between general smooth bounded pseudoconvex domains. In this talk, I will argue that this failure of global regularity is an underexplored PDE phenomenon that extends beyond the $\overline{\partial}$-Neumann problem. We will show that exact regularity, a stronger variant of global regularity, fails for a class of real sub-Laplacians on the boundaries of Diederich--Fornaess worm domains. The obstruction to exact regularity arises from the holonomy of certain foliations that occur naturally in this setting. We will conclude by formulating a few open problems and (shy) conjectures.

4. Tien Cuong Dinh (Singapore)

Two Unique Ergodicity theorems by Fornaess and Sibony

Abstract: Fornaess and Sibony proved two unique ergodicity theorems for the dynamical system associated with a complex Hénon map and for a generic foliation by Riemann surfaces in the projective plane. We'll present these results as well as the developments in these two areas of research.

5. John Erik Fornæss (NTNU)

Orbits and the Kobayashi metric

Abstract: Fatou components basically never have smooth boundary. This makes it difficult to estimate the boundary behaviour of invariant metrics. We consider the problem about Kobayashi density near the boundary of the backward orbit of a given point in a Fatou component. This is joint work in progress with Mi Hu and Feng Rong.

6. Purvi Gupta (IISc)

A complex-analytic characterization of Lagrangian immersions in $\mathbb{C}^n$ with transverse double points

Abstract. A compact set $\mathbb{C}^n$is said to be rationally convex if its complement in $\mathbb{C}^n$ is a union of complex hypersurfaces. Rational convexity endows a compact set with a Runge-type approximation property. A wider interest in rational convexity stems from some known symplectic characterizations of rational convexity for certain kinds of compact sets. For instance, a result by Duval--Sibony (1995) says that a compact totally real n-submanifold of $\mathbb{C}^n$ is rationally convex if and only if it is Lagrangian with respect to a Kahler form on $\mathbb{C}^n$. Such a characterization is lacking in the case of totally real immersions in $\mathbb{C}^n$. In this talk, we will first survey the known results in this context. We will then present a characterization of Lagrangian immersions admitting only transverse double points in terms of rational convexity. This is joint work with R. Sahu.

7. Hendrik Herrmann (Vienna)

Projective embeddings of CR manifolds via Toeplitz eigenspaces

Let $(X, T^{1,0}X)$ be a compact embeddable strictly pseudoconvex Cauchy–Riemann (CR) manifold. Motivated by Tian's classical approximation theorem for Kähler metrics, we study embeddings of X into complex projective space via weighted CR Kodaira maps constructed from eigenspaces of Toeplitz operators. We show that, as the eigenvalues tend to infinity, the pullback of the Fubini–Study metric under these maps admits an asymptotic expansion whose first two terms recover the underlying pseudohermitian structure. As an application, we prove that any Levi form on $(X, T^{1,0}X)$ can be approximated by zero divisors of CR functions. This is joint work with Chin-Yu Hsiao, George Marinescu and Wei-Chuan Shen.

8. Chin-Yu Hsiao (Taipei)

Localization of Bergman Kernels and the Cheng--Yau Conjecture on Real Analytic Pseudoconvex Domains

Abstract: In this work, we first establish the localization of the Bergman kernels for unbounded pseudoconvex domains near a D'Angelo finite type boundary point. Then we show that the Bergman metric of a bounded pseudoconvex domain with real analytic boundary is Einstein if and only if the domain is biholomorphic to the unit ball, thus contributing to an old conjecture of Cheng--Yau. This is a joint work with Xiaojun Huang and Xiaoshan Li.

9. Bernhard Lamel (Vienna)

Approximation of formal CR maps

Abstract : In this talk, we discuss some recent work with N. Mir, showing that formal CR maps between real-analytic hypersurfaces in two and three complex dimensions can be approximated (in the Krull topology) by convergent maps. These results are based on earlier work on divergent formal CR maps between real-analytic hypersurfaces and an analysis of their geometric consequences.

10. Andreea Nicoara (TCD)

Behavior of the q-type under Biholomorphic Mappings

Abstract: D'Angelo and Catlin q-types measure the order of contact of q-dimensional complex varieties with the boundary of a domain in $\mathbb{C}^n$ and are essential hypotheses for finding effective subelliptic estimates for the $\bar \partial$-Neumann problem on (p,q) forms. These two notions are the same for $q = 1$, and in that case, it is easy to prove that they are invariant under biholomorphic mappings. For $q > 1$, however, linear forms enter the definitions of both the D'Angelo and the Catlin q-types, and these forms are not necessarily mapped to linear forms by a biholomorphic map unless the map itself is linear. We define E_q another way of measuring the order of contact of q-dimensional complex varieties with the boundary of a domain in $\mathbb{C}^n$ which is quasi tautologically a biholomorphic invariant, and compare it to the D'Angelo and Catlin q-types. This is joint work with Vasile Brinzanescu (Simion Stoilow Institute of Mathematics of the Romanian Academy).

11. Mei-Chi Shaw (Notre Dame)

The Diederich-Fornaess Exponent and the $\bar{\partial}$-equation on Domains in CP^n

Abstract:

Diederich and Fornæss showed in 1977 that, for any bounded pseudoconvex domain Ω with C^2 boundary in a Stein manifold, there exist a positive constant $0 < \eta \leq 1$ and a defining function r such that $\hat{r} = -(-r)^\eta$ is plurisubharmonic on Ω. This result has many applications and extensions. In this talk we first discuss the consequences of such bounded plurisubharmonic exhaustion functions and its relations to solving the $\bar{\partial}$-equation on pseudoconvex domains in the complex projective space $\mathbf{CP}^n$. The proof depends on the existence of the Diederich and Fornæss exponent. For a domain in $\mathbf{CP}^n$ with C^2 boundary, this is proved by Ohsawa-Sibony and later extended by Harrington to Lipschitz domains. The Lipschitz condition cannot be dropped. In particular, we show that $\bar{\partial}$ does not have closed range for (2,1)-forms in L^2 on the Hartogs triangle in $\mathbf{CP}^2$. This is in sharp contrast to $\bar{\partial}$ on the Hartogs triangle in $\mathbf{C}^2$, where L^2 results have long been established by Hörmander.

The Hartogs triangle in $\mathbf{C}^2$ is the first example of a domain with no Stein neighborhood basis. The Diederich-Fornaess worm domains are smooth pseudoconvex domains in $\mathbf{C}^2$ with no Stein neighborhood basis. Hartogs triangles in $\mathbf{CP}^2$ are examples of non-smooth Levi-flat hypersurfaces. The Diederich-Fornaess exponent is also closely related to the non-existence of Levi-flat hypersurfaces in complex manifolds, a result proved earlier in a joint work with Siqi Fu and by Adachi-Brinkschulte independently.

12. Dmitri Zaitsev (TCD)

A new tower multitype invariant for non-pseudoconvex domains and applications to the compactness of the -Neumann operator.

Abstract: This report presents my recent joint work with A. Raich. The tower multitype was originally introduced to provide geometric conditions ensuring the compactness of the -Neumann operator on pseudoconvex domains. For higher-degree forms, however, the relevant estimates involve partial traces of the Levi form rather than the full Levi form, suggesting that positivity of the Levi form may be relaxed to positivity of its traces. In this context, we introduce a new tower multitype based on subbundles of exterior powers of the tangent bundle, rather than the tangent bundle itself. Our main result shows that finiteness of this multitype implies compactness of the -Neumann operator.