



Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin

Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JF Geography and Geoscience

Michaelmas Term 2018

MAU11001: Mathematics Statistics and Computation

Tuesday, December 11

Exam Hall

14.00 — 16.20

Prof. S. Ryan, Prof. A. Ramos

Instructions to Candidates:

Credit will be given for the best ONE question answered in Section A and the best TWO questions answered in Section B.

Use a different answer book for each section.

Formulae and log tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination.

Please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

Section A

Credit will be given for the best question answered in this section.

1. Consider the 2×2 matrix, A

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

- (a) [4 marks] By calculating the determinant of A show that it is invertible.
- (b) [10 marks] Use Gauss-Jordan elimination to find the inverse, A^{-1} of A and verify that your answer is correct.
- (c) [6 marks] Use the inverse you calculated to solve the system of equations

$$2x + y = 3,$$

$$x + 2y = 7.$$

2. Consider an $n \times n$ matrix, A .

- (a) [4 marks] Write down its characteristic equation and identify each term in the equation.
- (b) [12 marks] Given

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}.$$

Determine the eigenvalues and eigenvectors of A .

- (c) [4 marks] If the 2×2 -matrix A given above is a Leslie matrix and $P = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ describes a species population in year N , determine the population in year $N + 1$.

Section B

Credit will be given for the best 2 questions answered in this section.

3. Consider the function

$$f(x) = x^4 + 3x^3 + 2x^2$$

(a) [5 marks] Determine the domain of the function and its roots.

(b) [5 marks] Determine the asymptotes of the function.

(c) [5 marks] Determine the local maxima and minima.

(d) [5 marks] Draw a sketch of the function

4. Consider the function

$$f(x) = \frac{x^2 + 3x + 2}{x + 1}.$$

(a) [5 marks] Determine

$$\lim_{x \rightarrow -1} f(x).$$

(b) [5 marks] Determine

$$\frac{df(x)}{dx}.$$

(c) [5 marks] Determine

$$\int f(x) dx$$

(d) [5 marks] Determine

$$\frac{d}{dx} \int_0^x f(t) dt.$$

5. A piece of string of length 1m is divided in two parts of sizes x and $1 - x$ in order to make two rectangles. One with one side three times larger than the other and the other with one side two times larger than the other (see figure 1). How do we have to choose x to make the sum of the areas of the two rectangles minimum? [20 marks]

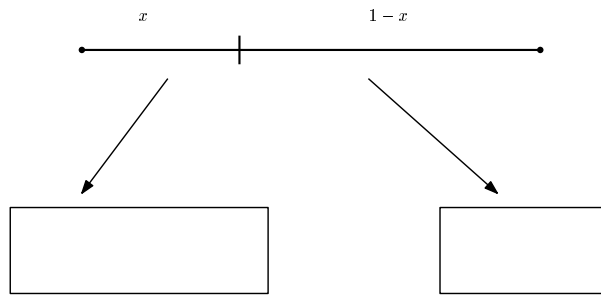


Figure 1: Problem 5