Q1

a) $\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

b)

$$(A|I) = \left(\begin{array}{cc|cc} 3 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_1/3} \left(\begin{array}{cc|cc} 1 & 2/3 & 1/3 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 - 2R_1} \left(\begin{array}{cc|cc} 1 & 2/3 & 1/3 & 0 \\ 0 & -1/3 & -2/3 & 1 \end{array}\right)$$

$$\xrightarrow{-3R_2} \left(\begin{array}{cc|cc} 1 & 2/3 & 1/3 & 0 \\ 0 & 1 & 2 & -3 \end{array}\right)$$

$$\xrightarrow{R_1 - \frac{2}{3}R_2} \left(\begin{array}{cc|cc} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & -3 \end{array}\right) \quad \underbrace{\begin{matrix} -1 & 2 \\ 2 & -3 \end{matrix}}_{A^{-1}}$$

CHECK

$\begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

c) Since $Ax = b \Rightarrow A^{-1}Ax = A^{-1}b$ ie $x = A^{-1}b$

$x = \begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ ie $\left.\begin{matrix} x = -1 \\ y = 3 \end{matrix}\right\}$ CHECK BY SUBS.

Q2

a leslie mx is o.t.f. $\begin{pmatrix} \text{fecundity of hatchlings} & \text{fecun. of adults} \\ \text{survivability hatchling} & \text{surv. of adults} \end{pmatrix}$

a) 30% hatchlings survive

b) 0% adults survive

c) 2.3 = avg. number of offspring

d) EIGENVALUES from $\det(A - \lambda I) = 0$

$$\det \begin{pmatrix} 1-\lambda & 2.3 \\ 0.3 & -\lambda \end{pmatrix} = (1-\lambda)(-\lambda) - 2.3(0.3) = 0$$

$$\Rightarrow \quad \lambda^2 - \lambda - 0.69 = 0 \quad \Rightarrow \quad \lambda_1 = 1.47, \quad \lambda_2 = -0.4695.$$

e) $\lambda = 1.47$ and e'vec = stable age pop. from $Av = \lambda v$

$$\begin{pmatrix} 1 & 2.3 \\ 0.3 & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 1.47 \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \Rightarrow$$

$v_1 + 2.3v_2 = 1.47v_1$

$4.9v_2 = v_1$

$0.3v_1 = 1.47v_2$

$v_1 = 4.9v_2$

stable age pop = $\begin{pmatrix} 4.9v_2 \\ v_2 \end{pmatrix}$

for $v_2 \in \mathbb{R}$.

$\lambda_1 = 1.47 > 1 \Rightarrow$ pop. growth.

Q3. a) <u>BINOMIAL EXPT.</u>

- trial repeated n times
- each trial independent.
- each trial has only 2 outcomes : success or failure
- probability of success is constant.
- goal of exp. is to count successes.

b)
i) NO — answer/result not success or failure

ii) YES — meets all criteria above

iii) ~~NO~~ YES — $n = 1$; success = roll 5.

iv) NO — not counting "success"

v) NO — 'n' not fixed

vi) NO — without replacement draws not independent.

c) Want $P(X = 3)$ where X = number of people with A

and $p = 0.1 \Rightarrow q = 0.9$; $n = 5$.

$$P(X=3) = \binom{5}{3}(0.1)^3(0.9)^{5-3} = 0.0243$$

ie approx 2.4% prob.

Q4

a) show $\int_a^b f(x) = 1$

$$\int_0^1 (2-2x)dx = 2x - \frac{2}{2}x^2 \Big|_0^1 = 1$$

b) $$P(X > 0.5) = \int_{0.5}^1 f(x)\,dx$$

$$= \int_{0.5}^1 2-2x \; dx = 2x - x^2 \Big|_{0.5}^1$$

$$= (2-1) - (1 - 0.25)$$

$$= \underline{0.25}$$

c) $$\mu = \int_a^b x f(x)\,dx = \int_0^1 2x - 2x^2 \, dx$$

$$= x^2 - \frac{2}{3}x^3 \Big|_0^1 = 1 - \frac{2}{3} = \underline{\underline{\frac{1}{3}}}$$

$$\sigma^2 = Var(X) = \int_a^b (x-\mu)^2 f(x)\,dx$$

$$= \int_a^b x^2 f(x)\,dx - \mu^2$$

$$\int_0^1 x^2 f(x)\,dx = \int_0^1 2x^2 - 2x^3 = \frac{2}{3}x^3 - \frac{2}{4}x^4 \Big|_0^1$$

$$= \frac{2}{3} - \frac{1}{2}$$

$$= \frac{1}{6}$$

$$\Rightarrow \quad SD(X) = \sqrt{Var(X)} = \sqrt{\left(\frac{1}{6} - \frac{1}{9}\right)} = \sqrt{\frac{1}{18}}$$